Photon Localization and mesoscopic effects in cold atoms

- Multiple scattering of photons
- The coherent backscattering effect
- Phase coherence times
- Strong disorder : Anderson phase transition *vs*. photon localization
- Effective Hamiltonian and escape time distribution

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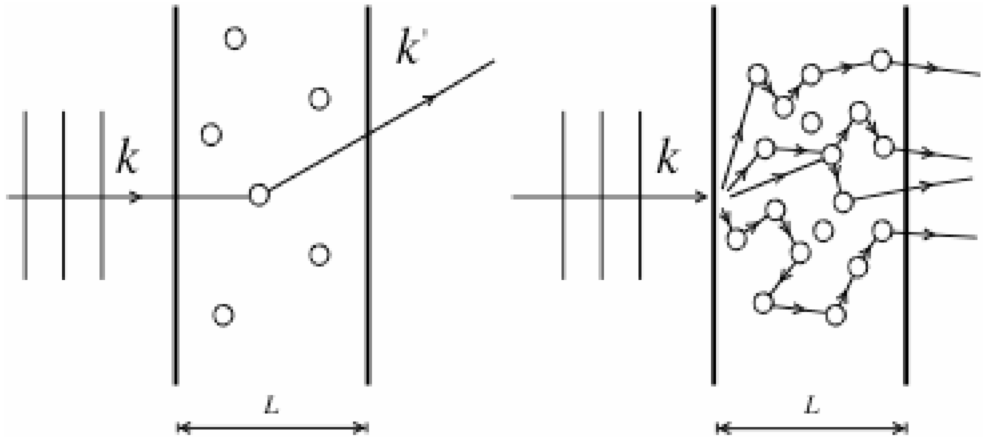
Multiple scattering



Characteristic lengths:

Wavelength: λ

Elastic mean free path: $l_e = 1/n\sigma \gg n^{-1/d}$

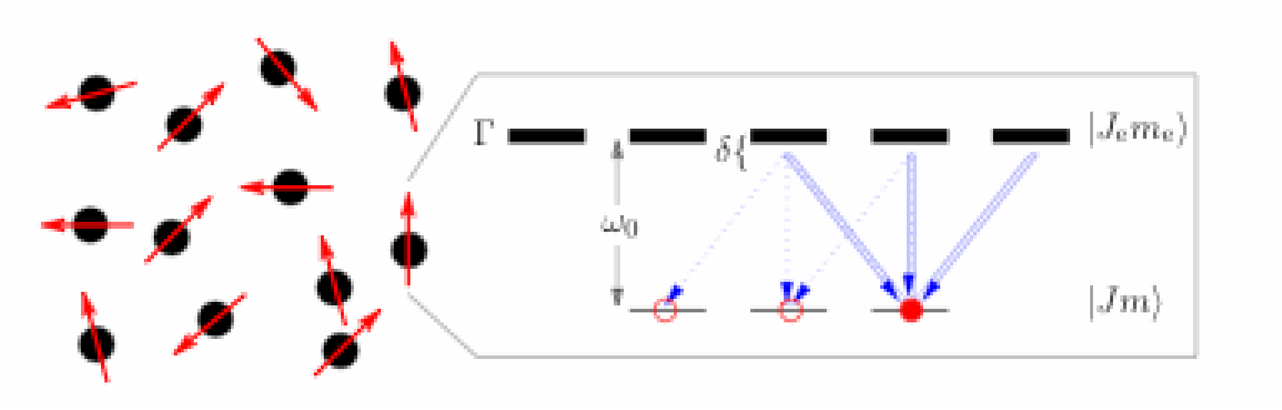
σ= scattering cross section

n= density of scatterers

Weak disorder : $\lambda \ll l_e$

$\Rightarrow$ Independent scattering events

Light scattering in a dilute cloud of cold atoms



- Photon-atom interaction: dipolar interaction $V = -D \cdot E(r)$

A degenerate atomic dipole transition (J, J_e) allows

Rayleigh scattering $(m' = m)$ and Raman scattering $(m' \neq m)$

- Average light propagation in a cold atomic gas:
- Trace over the positions of the atoms
- Trace over the quantum numbers m with a scalar atomic density matrix.
- Dilute medium $(n\lambda^3 \ll 1)$ of weak and resonant scatterers $(l_e \gg \lambda)$

Scattering cross section
elastic mean free path

The resonant scattering cross section σ is related to the elastic mean free path l_e by $l_e = 1/n\sigma$

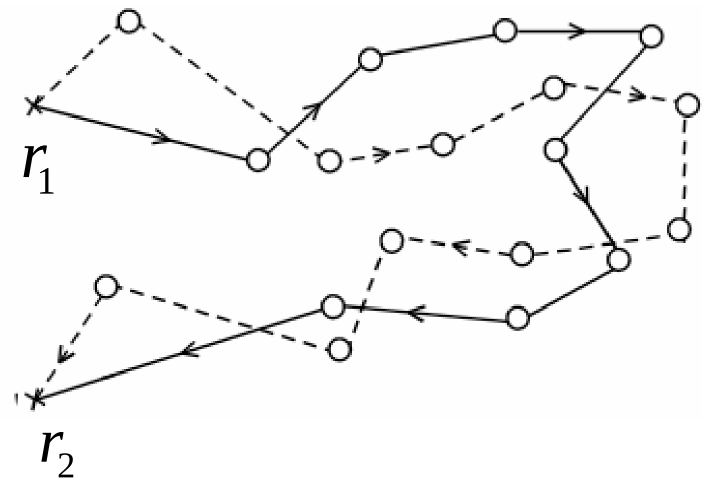
So that:

$$\frac{\lambda}{l_e} = nA_{JJ_e}\frac{3\lambda^3}{2\pi}\frac{1}{1+(2\delta/\Gamma)^2}$$

With

$$A_{JJ_e} = \frac{1}{3}\frac{2J_e+1}{2J+1}$$

For ^{85}Rb at resonance and for weak disorder, *i.e* for a density $n \simeq 6\times10^{10}\,cm^{-3}$ we have $l_e \simeq 133\mu m$

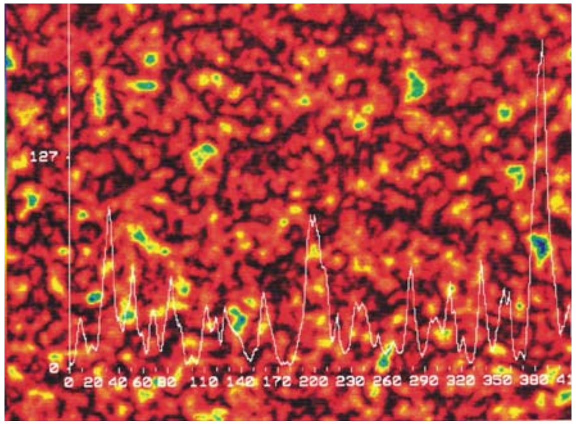


- Measure intensities *i.e.* product of complex amplitudes.

$$A(\vec{k},\vec{k}') = \sum_{r_1,r_2} f(r_1,r_2)\exp[i(\vec{k}\cdot r_1 - \vec{k}'\cdot r_2)]$$

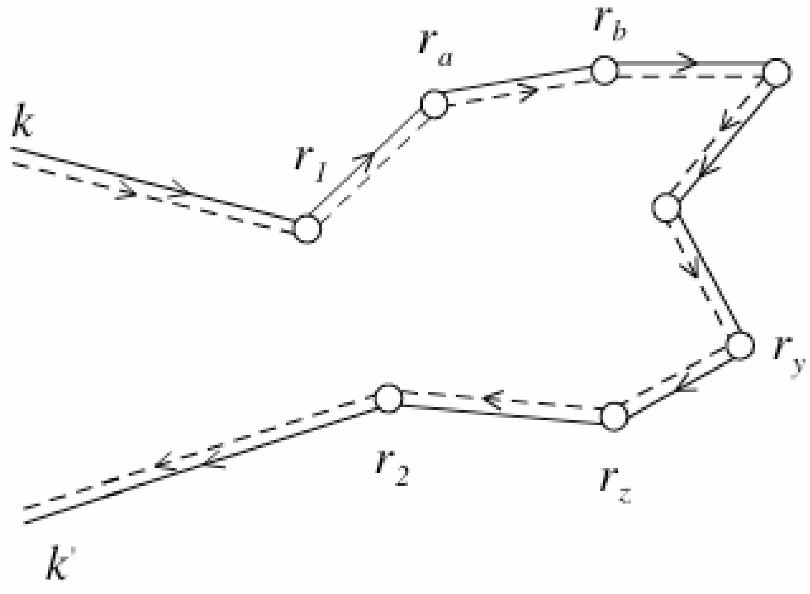
with $$f(r_1,r_2) = \sum_j a_j \exp(i\delta_j)$$

Sum over all paths j



- Before averaging: Speckle pattern (full coherence)
- Configuration average: most of the contributions vanish because of large phase differences.
- **What are the remaining terms?**

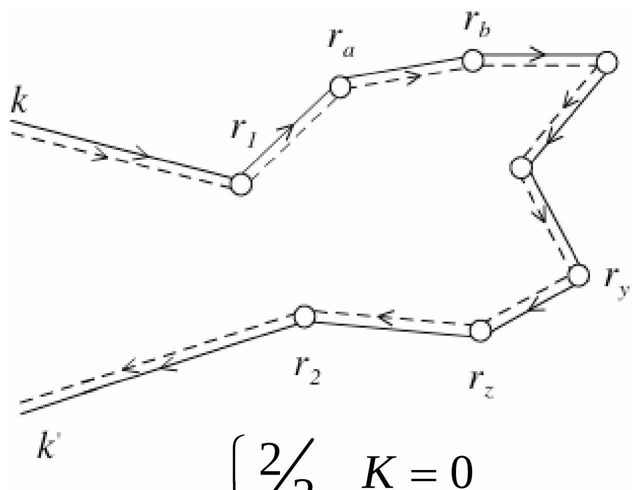
Average intensity: $I_{cl} = \overline{\left|A(\vec{k},\vec{k}')\right|^2} = \sum_{r_1,r_2} \overline{\left|f(r_1,r_2)\right|^2}$



Phase independent : classical description **DIFFUSON**

Interference effects do not survive average multiple scattering:
Success of Radiative transfer or Drude theory for metals.
Accounts for a large number of phenomena in multiple scattering of photons by atoms : Brossel group, Barrat, Omont

Modes of the diffuson



$$b_K = \begin{cases} 2/3 & K=0 \\ 1/3 & K=1 \\ 7/15 & K=2 \end{cases}$$

$$\lambda_K = 3A_{JJ_e}(2J_e+1)\begin{Bmatrix} 1 & 1 & K \\ J_e & J_e & J \end{Bmatrix}^2$$

τ_K : depolarization times

- Incoherent contribution to the average intensity.

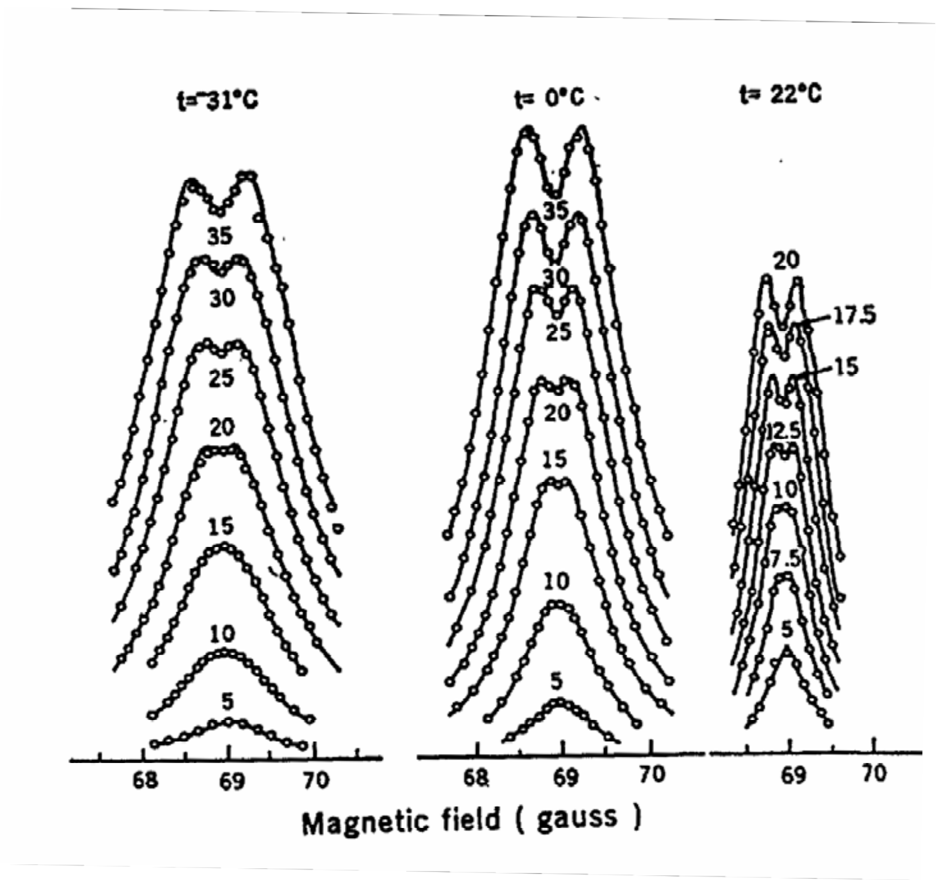
Fourier transform of $\Gamma(r_1, r_2)$ involves 3 modes

$$\Gamma_K(q) = \frac{1}{Dq^2 + 1/\tau_K} \quad (K = 0,1,2)$$

each of total spin equal to K= 0,1,2.

$$\Gamma\tau_K = \frac{b_K\lambda_K}{\frac{2}{3}A_{JJ_e} - b_K\lambda_K}$$

Narrowing of the double resonance lines in a magnetic field

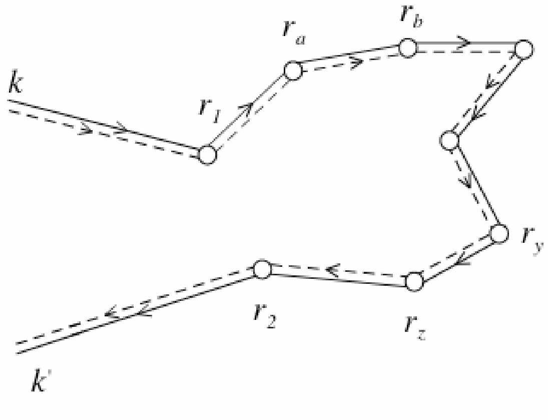


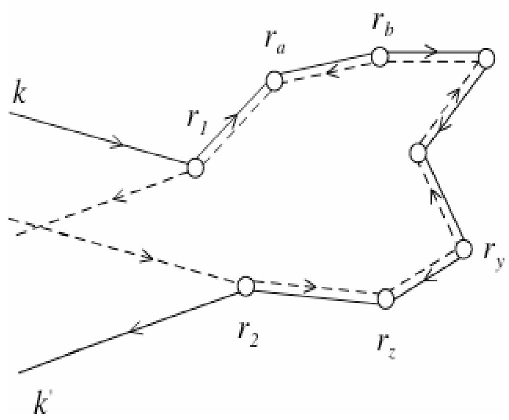
- Hanle effect: scattered light by a vapor in a magnetic field.
- Narrowing of the lines at larger densities, *i.e.* a longer Zeeman coherence and magnetic moment.
- Transfer of coherence between atoms in multiple scattering
- Shift of the lines (Omont)

Linewidth: $^{JJ}\tau = \Gamma^{-1}\left(1 + \frac{7}{15}\Gamma\,\tau_2\right)$

Brossel group, Barrat

Cooperon



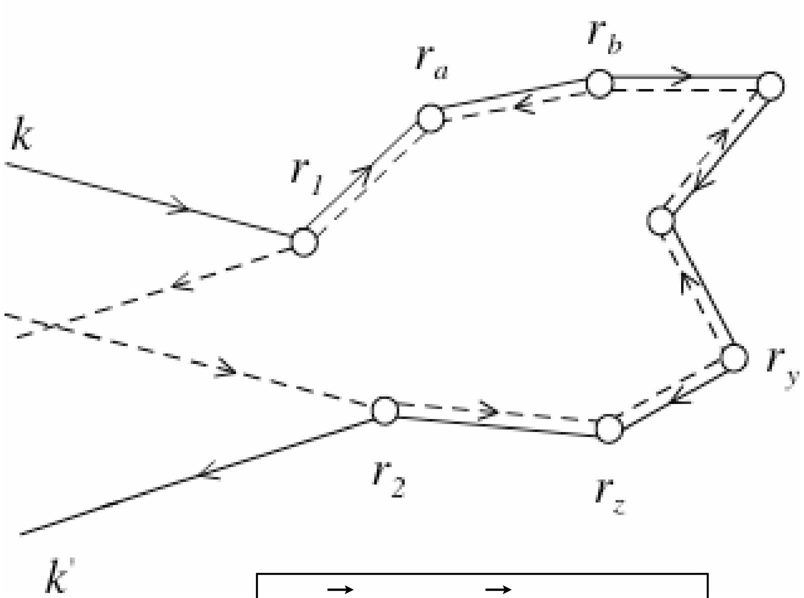


Consider the same amplitude but time reversed
Cooperon

Equivalent of a Young interferometer for multiple scattering

Coherent backscattering

E.A and R. Maynard (1986)



$$I(\vec{k}' = -\vec{k}) = 2I_{cl}$$

- Total average intensity

the sum makes the interference term vanishing except for:

Coherent backscattering:

$$\vec{k} + \vec{k}' \approx 0$$

last interference term to survive on average for a weak disorder.

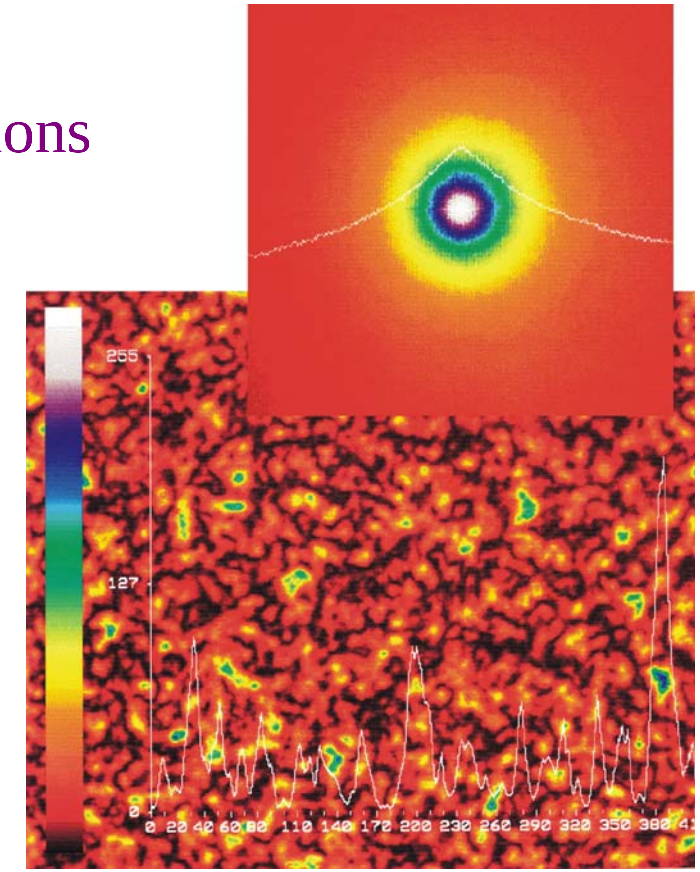
- $r_1 = r_2$ closed loops

weak localization

$$I = \overline{\left|A(\vec{k},\vec{k}')\right|^2} = \sum_{r_1,r_2} \overline{\left|f(r_1,r_2)\right|^2}\left[1 + \exp\left\{i(\vec{k}+\vec{k}')\cdot(r_1 - r_2)\right\}\right]$$

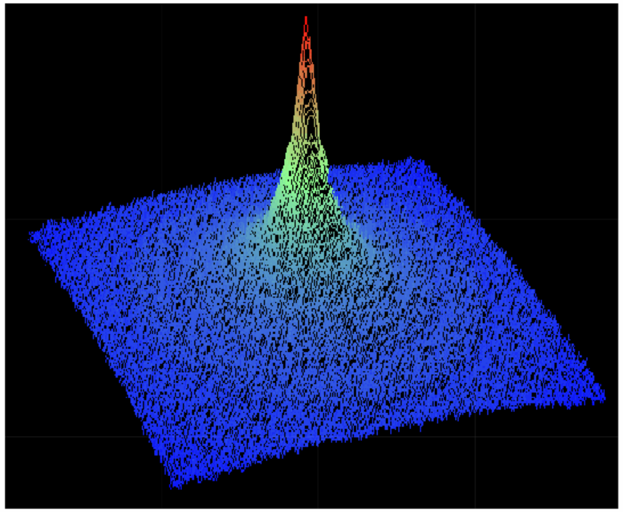
Speckle and coherent backscattering

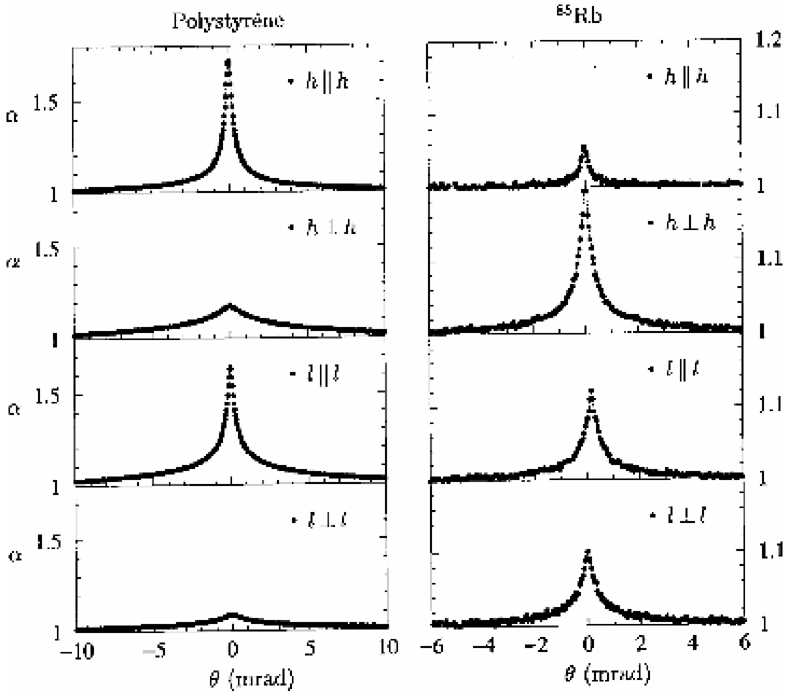
G. Maret (2000)
Colloidal suspensions



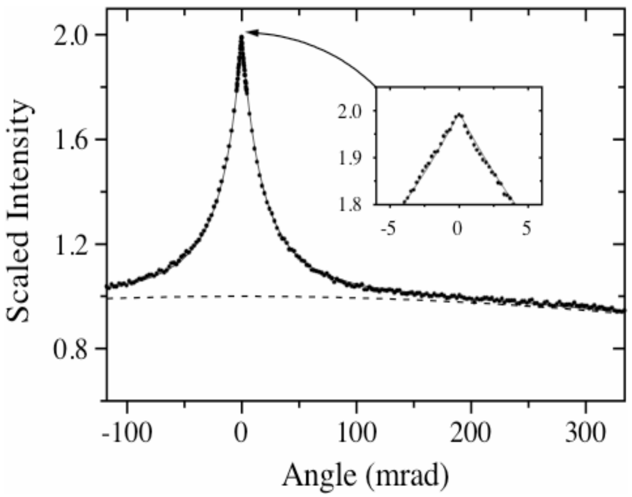
Coherent backscattering of light by cold atoms (Rb85)

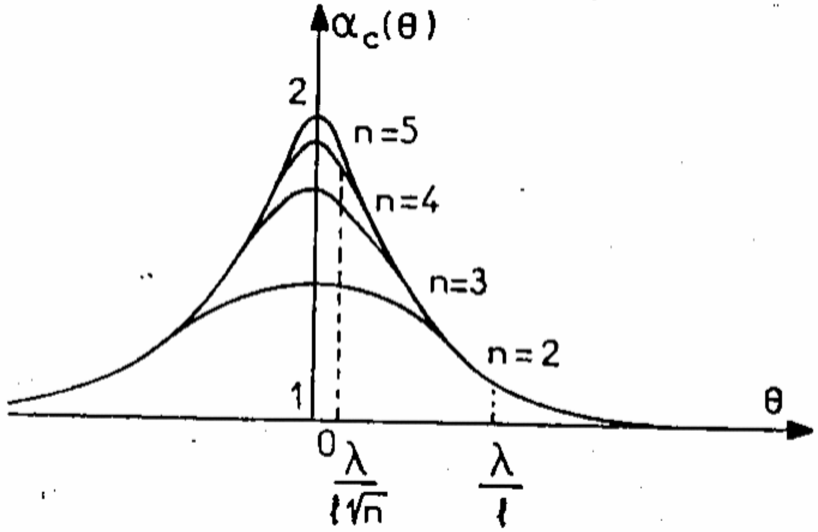
Kaiser et al. 2000





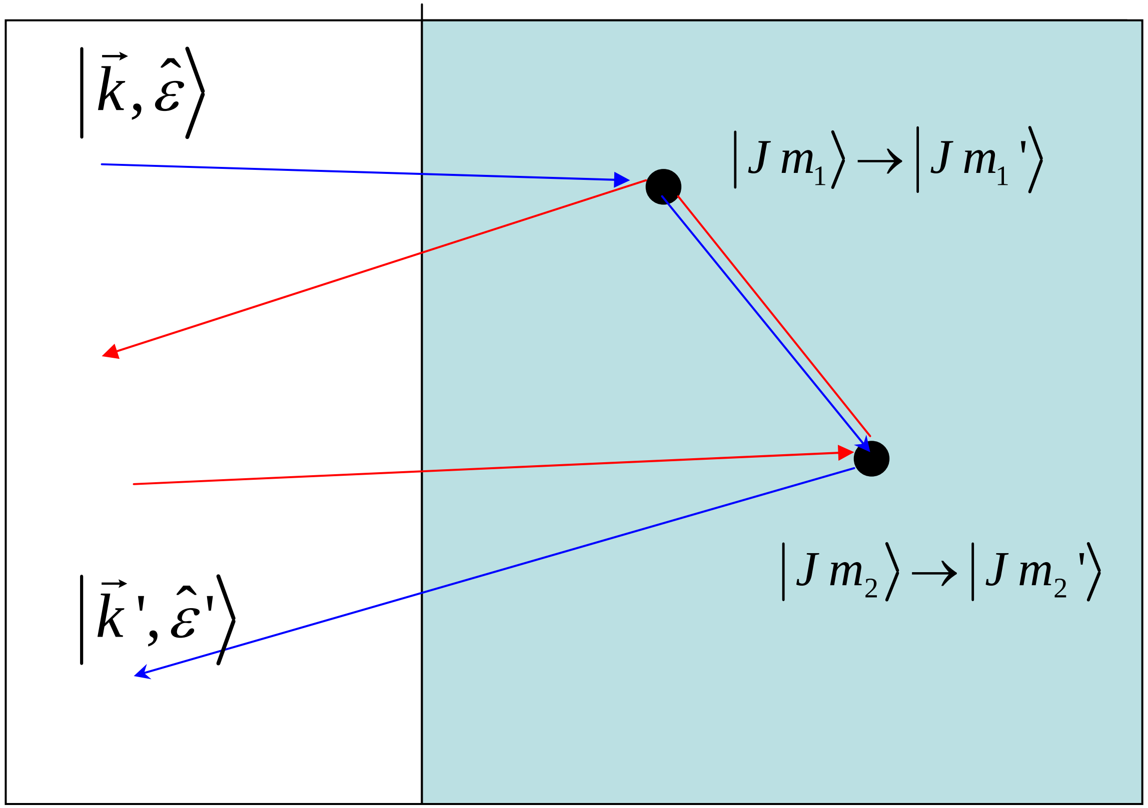
Coherent backscattering cone





- The coherent backscattering cone is the superposition of coherent contributions of lengths nl_e in a cone of angular aperture $\lambda / l_e\sqrt{n}$
- In the presence of decoherence described by a phase coherence time τ_φ, trajectories of lengths larger than L_φ do not contribute ⇒ the CBS is rounded off at small angles (interferometer)

Reduced interference contrast



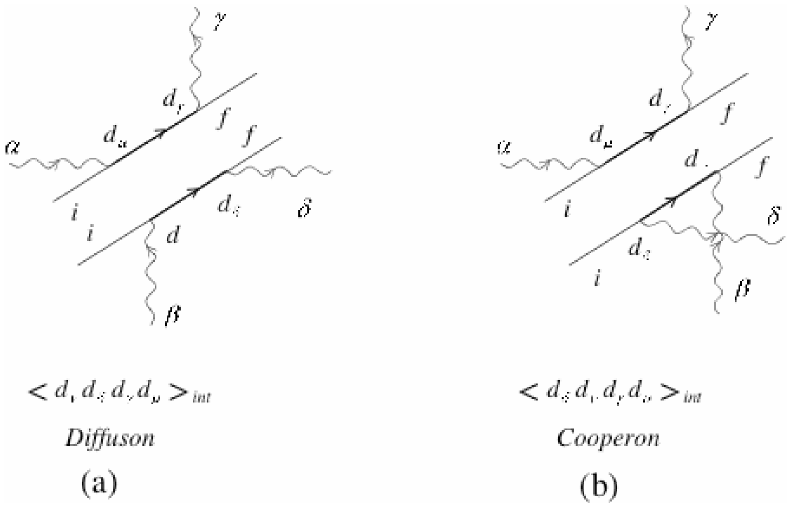
Coherent backscattering (CBS): amplitudes of direct and reversed paths interfere constructively in backscattering($\vec{k}' = -\vec{k}$)

Reciprocity: CBS amplitudes are equal for

$$\vec{k}' = -\vec{k} \quad \hat{\varepsilon}' = \hat{\varepsilon}^* \; and \; \{m'\} = \{-m\}$$

But generally, $\{m'\} \neq \{-m\}$

Sources of decoherence



$$\lambda_K = 3(2J_e+1)\begin{Bmatrix} 1 & 1 & K \\ J_e & J_e & J \end{Bmatrix}^2$$

diffuson

$$\chi_K = 3(2J_e+1)\begin{Bmatrix} 1 & J_e & J \\ 1 & J & J_e \\ K & 1 & 1 \end{Bmatrix}$$

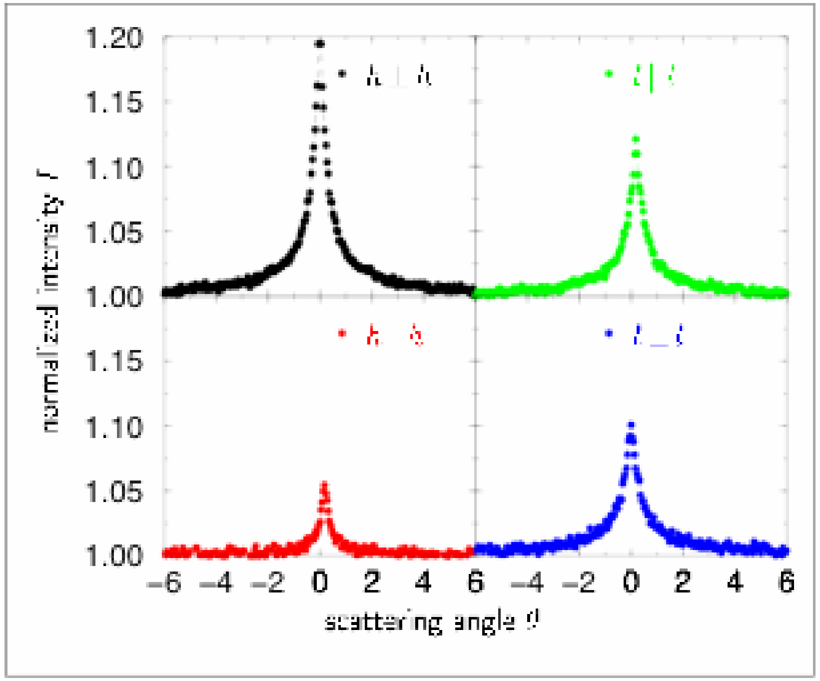
cooperon

Phase coherence time $\tau_\varphi(K)$ is defined through the decay of the Coherent intensity with respect to the incoherent part.

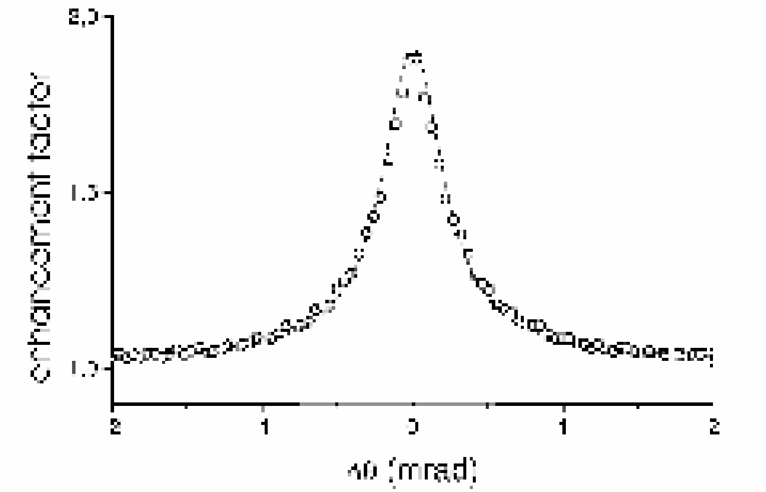
$$\Gamma\tau_\varphi(K) = \frac{b_K}{1/\chi_K - 1/\lambda_K}$$

The classical Rayleigh scattering $J=0$ corresponds to $\lambda_K = \chi_K = 1$ and thus to the absence of dephasing: $\tau_\varphi \to \infty$

Measurement of the phase coherence time



Rb $J = 3, J_e = 4$



Sr $J = 0, J_e = 1$

Comparison of CBS by laser cooled atoms without and with internal degeneracy shows a drastic reduction of interference even for preserved helicity

Strong disorder

Onset of a photon localization transition: for a critical amount of disorder (density of atoms) : phase transition from delocalized to localized photon states.

Ioffe-Regel criterion: $\lambda \approx l_e$ and $l_e = 1/n\sigma$

For resonant atom-photon scattering:

$$\frac{\lambda}{l_e} = nA_{JJ_e} \frac{3\lambda^3}{2\pi} \frac{1}{1+(2\delta/\Gamma)^2}$$

with $A_{JJ_e} = \frac{1}{3}\frac{2J_e+1}{2J+1}$

At resonance $\delta \approx 0$ so that $\frac{\lambda}{l_e} \approx n\lambda^3$

Cold vapor of ^{85}Rb : $\lambda / l_e \approx 10^{-4}$ (far from localization)

Bose-Einstein condensate: $n \approx 10^{14}\, cm^{-3}$ we have: $\lambda / l_e \approx 3.4$

Collective effects and dipole-dipole resonant interation

Dipolar atoms are not independent scatterers.
There are long range dipole-dipole interactions.
Moreover, when two resonant scatterers are close enough, collective states appear (Dicke states) that change substantially the nature of the localization transition.

Characterize the onset of localization transition by mean of the **distribution P(t) of escape times.**

Effective Hamiltonian

Atoms = collection of resonant two-level systems:

Ground state: $|g_i\rangle$ Excited state: $|e_i\rangle$

$$H_{\text{int}} = -i\hbar\frac{\Gamma}{2}\sum_{i=1}^{N}|e_i\rangle\langle e_i| - \hbar\frac{\Gamma}{2}\sum_{i\neq j}\frac{e^{ik|r_i - r_j|}}{k|r_i - r_j|}d_i^{+}d_j^{-}$$

with $d_i^{+} = d|e_i\rangle\langle g_i|$ and $d_i^{-} = (d_i^{+})^{\dagger}$

Diagonal elements: spontaneous emission of isolated atoms

Off-diagonal terms: modification of the spontaneous emission due to collective effects and dipole-dipole resonant interaction.

Distribution of escape times

Probability $\pi(t)$ that a photodetector placed outside the atomic cloud will detect a photon at time t .

$$\pi(t) = \Gamma \sum_{i,j} \left\langle \frac{\sin(k\, r_{ij})}{k\, r_{ij}} d_i^{+}(t) d_j^{-}(t) \right\rangle$$

Probability to detect a photon between times 0 and t : $P(t) = \int_0^t \pi(t')\,dt'$

Averaging over the positions of the atoms :

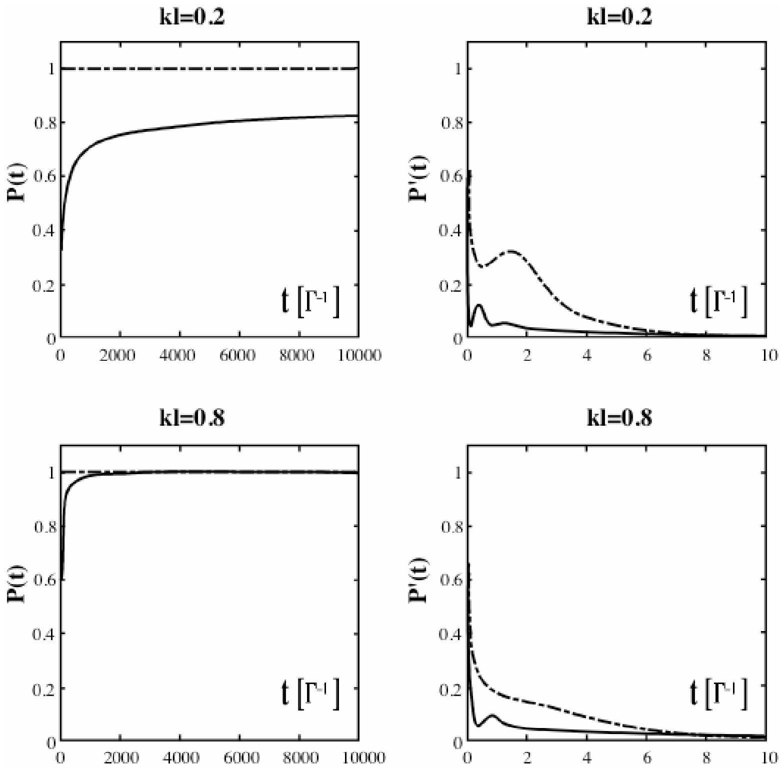
$$\bar{P}(t) = 1 - \overline{\sum_i \left| \langle i:e | \exp(-i\, H_{\text{int}} t / \hbar | \Psi_i \rangle \right|^2}$$

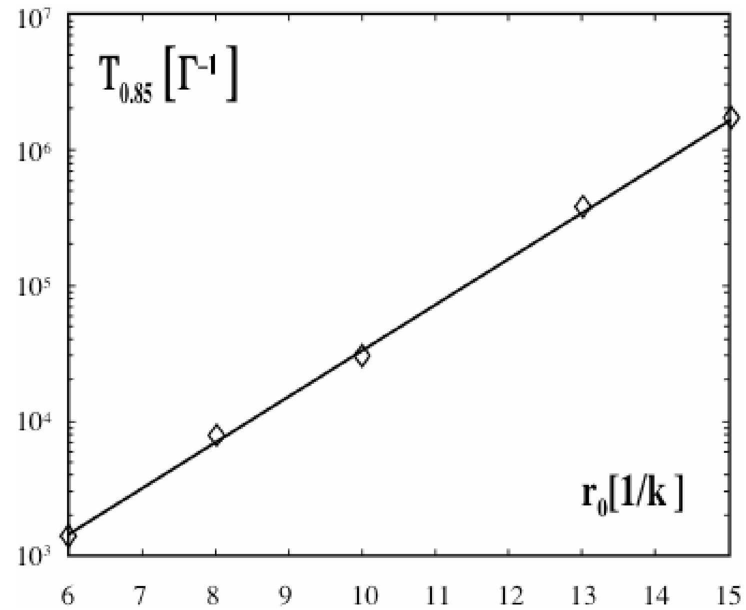
$|i:e\rangle$ states for which only one atom is in the excited state

$$|\Psi_i\rangle = |g, \phi : (N-1); e, \chi : 1\rangle$$

Initial state (Thomas-Fermi approx. for the BEC)

Distribution of escape times



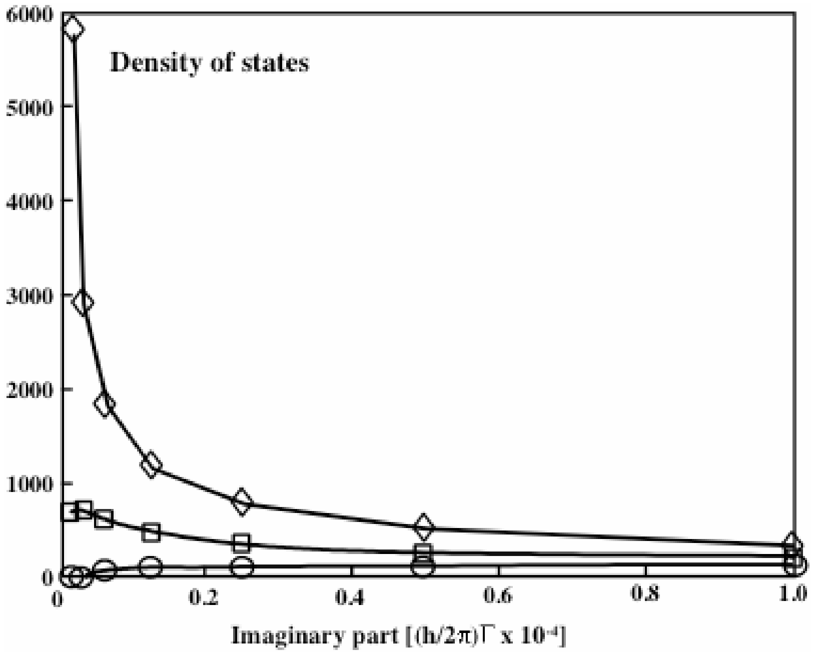


- Strong distinction between an initially localized or initially delocalized photon mode.
- For an initially localized mode, the typical escape time varies exponentially with the size r_0 of the atomic cloud.
- In the weak disorder limit, the typical escape time varies as r_0^2

Non hermitic effective Hamiltonian

$|r_n\rangle$ and $\varepsilon_n = \nu_n - i\gamma_n$ eigenvectors and (complex) eigenvalues of H_{int}

$$\bar{P}(t) = 1 - \overline{\sum_n e^{-2\gamma_n t/\hbar} |o_n|^2}$$



- Relate the escape times to the imaginary part γ_n of the eigenvalues of the non hermitic Hamiltonian H_{int}

$$o_n = \frac{\langle r_n^* | \Psi_i \rangle}{\langle r_n^* | r_n \rangle}$$

Measures the overlap between the eigenstates of H_{int} and the initial state.

Summary and future directions

- Study of coherent properties of the photon-atom system in the multiple scattering limit using the coherent backscattering effect.
- Cold atomic gases represent an ideal system to reach the onset of photon localization transition (Anderson).
- The critical disorder corresponds to the density of a BEC.
- The existence of additional degrees of freedom change the nature of the transition (critical exponents…)
- The coherent backscattering effect can be used to build sensitive interferometers using the Sagnac effect (precise rotation measurements)