Making Sense of Teaching, and Working to Improve It

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My Goal for Today:

To lay out a framework, and a program, for professional development aimed at helping students become mathematical sense-makers.

Outline

1. On mathematical sense-making
2. How the US Context may support it.
3. On decision-making and change
4. On productive classrooms - The TRU Math Scheme as an intellectual framework
5. A tool - Formative Assessment Lessons
6. A tool - The question version of TRU Math
7. A grand plan for change
8. Discussion

1&2
Some Quick Thoughts on Goals and Context

Goals Matter.

What are your goals for students' mathematics learning? Content mastery, problem solving, developing mathematical habits of mind? Or...?

Context Matters.

How are your goals embodied in teacher preparation, curriculum, student testing, teacher evaluation?

Do you know the phrase

W Y T I W YG

?

Here are some examples from California.

High-stakes tests focused on basic skills:

23 **What is the *y*-intercept of the graph of $4x + 2y = 12$?**

A -4

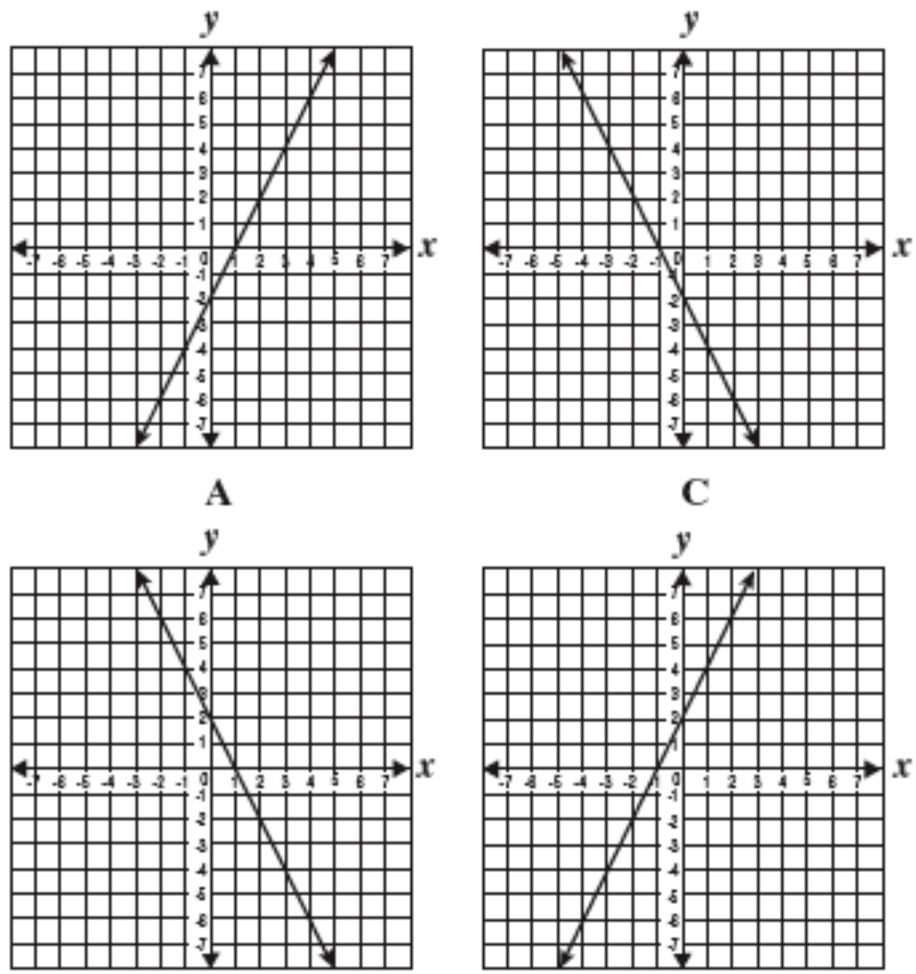
B -2

C 6

D 12

CSA00239

25 Which *best* represents the graph of $y = 2x - 2$?



A C

B D

Because the scores mattered so much - student, teacher, and school evaluations depend on them – such skills-oriented, conceptually impoverished mathematics is what instruction focused on.

But, this is about to change.

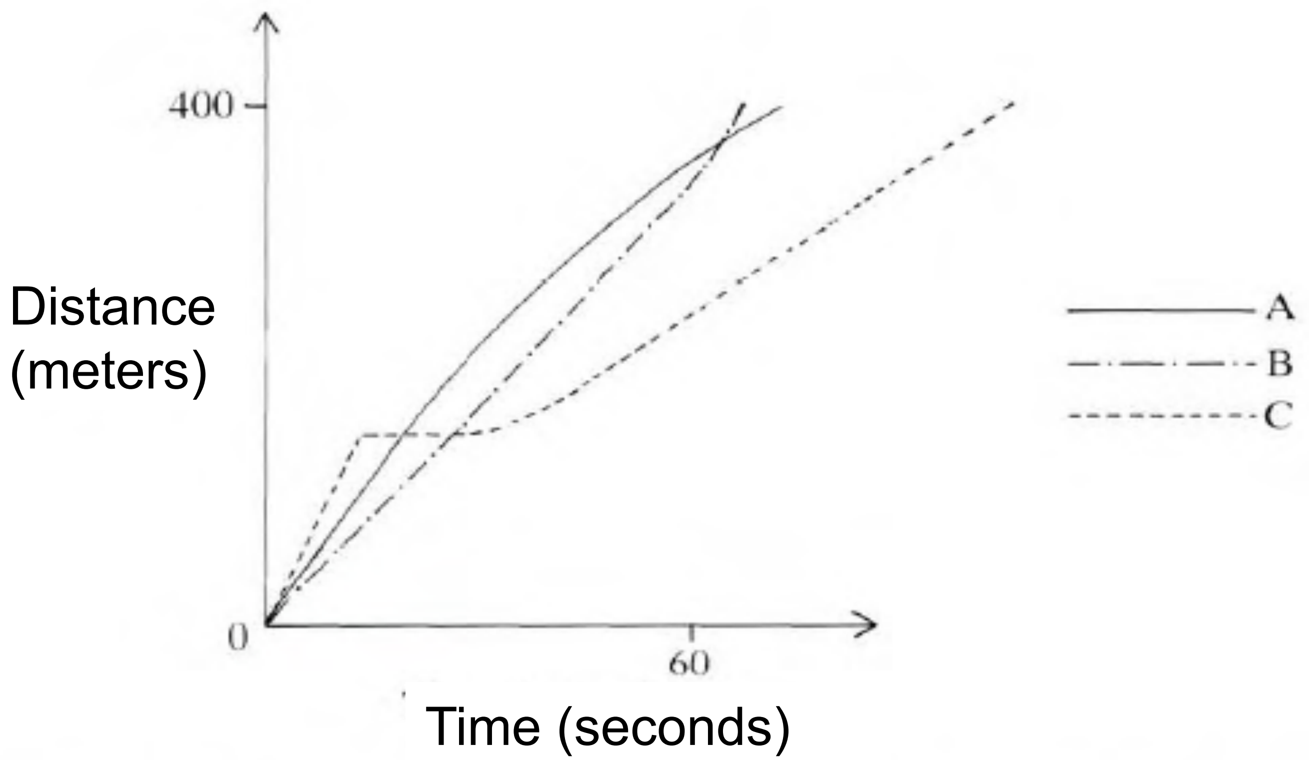
COMMON CORE STATE STANDARDS FOR

Mathematics

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Here is a problem of the type I hope will be on the new national exams:

Hurdles Race



This is a rough sketch of 3 runners' progress in a 400 meter hurdle race. Imagine that you are the race commentator. Describe what's happening as carefully as you can. You do not need to measure anything accurately.

Think of the Content involved:

- Interpreting distance-time graphs in a real-world context
- Realizing "to the left" is faster
- Understanding points of intersection in that context (they're tied at the moment)
- Interpreting the horizontal line segment
- Putting all this together in an explanation

Think of the Practices involved:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments…
- Model with mathematics…

25% Sale, Part 1

In a sale, all the prices are reduced by 25%. Julie sees a jacket that cost $32 before the sale. How much does it cost in the sale?

25% Sale, Part 2

In the second week of the sale, the prices are reduced by 25% of the previous week's price.

In the third week of the sale, the prices are again reduced by 25% of the previous week's price.

In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

Alan says that after 4 weeks of these 25% discounts, everything will be free. Is he right? Explain your answer.

Again:
Core content, central practices.

Want to see more?

Check out Consortium specs;

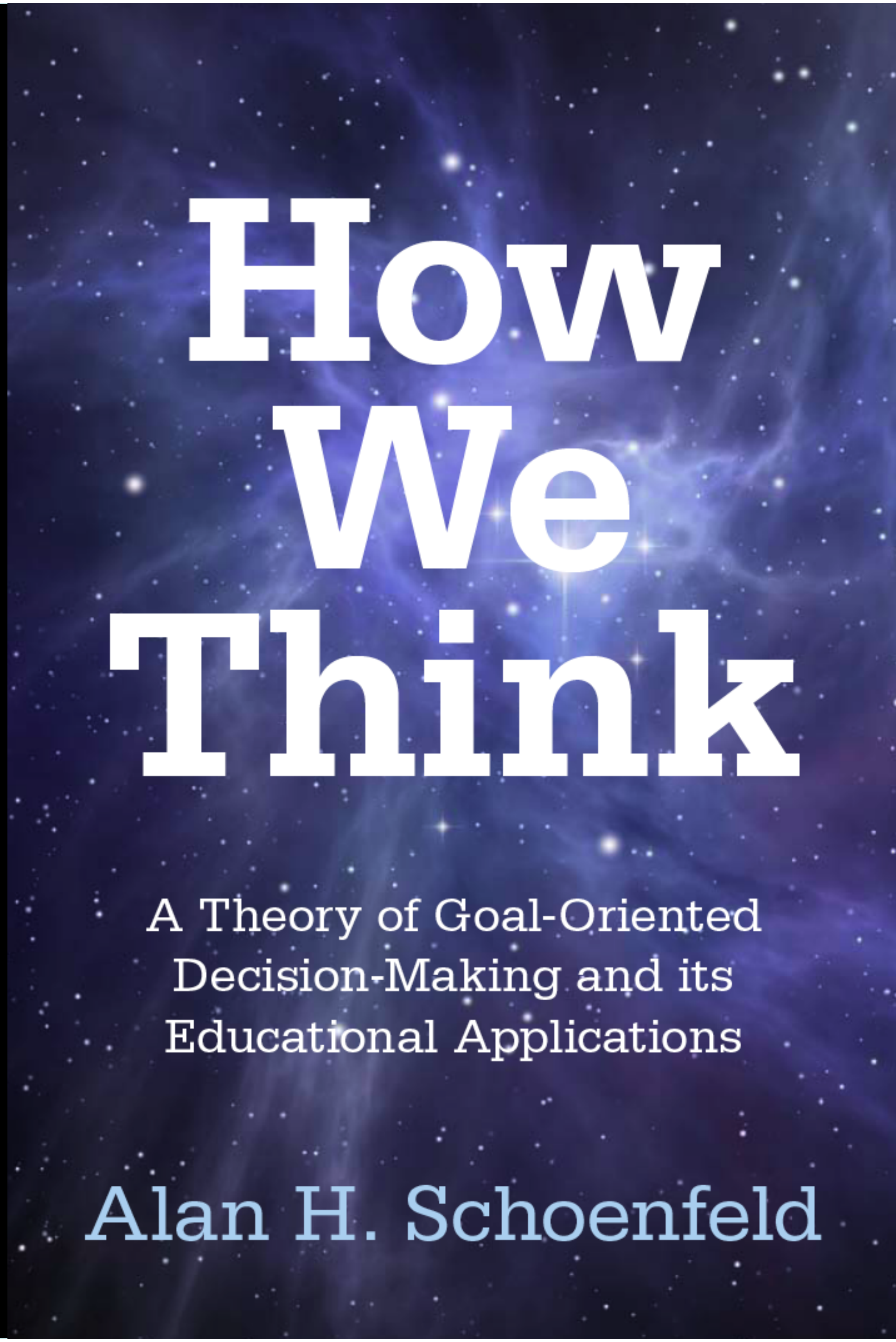
look at the

Mathematics Assessment Project

(google the name).

3.
On decision-making and change

(25 years of research in 3 minutes)


How We Think
A Theory of Goal-Oriented Decision-Making and its Educational Applications
Alan H. Schoenfeld

A teacher's decision making - *anyone's* decision making - is a function of that person's:

- Knowledge and Resources
- Beliefs and Orientations
- Goals

The first two of these develop very slowly (and the ability to implement the 3rd is a function of the others).

That’s why developing expertise in any area that calls for significant thinking and decision making (e.g., surgery, chess, electronic trouble-shooting, *teaching*) takes from 5000 to 10,000 hours of practice and reflection.

This means:

There are *no quick fixes* in professional development. If we want teachers to be able to teach in ways that produce students who are powerful thinkers, we will need to invest TIME as well as effort in helping them develop as teachers. And we need good tools as well.

4.
On productive classrooms – The TRU Math Scheme as an intellectual framework

We’ll watch two videos, to get a sense of the range of classroom activities we need to capture in a coding scheme:

A. The TIMSS U.S. Geometry Video
B. The “Border Problem” taught by Cathy Humphreys

A.
The TIMSS U.S. Geometry Video

This video exemplifies what we call IRE sequences:

Initiation (teacher asks a question)

Response (from the student)

Evaluation from the teacher.

A key feature: bite-size pieces of knowledge!

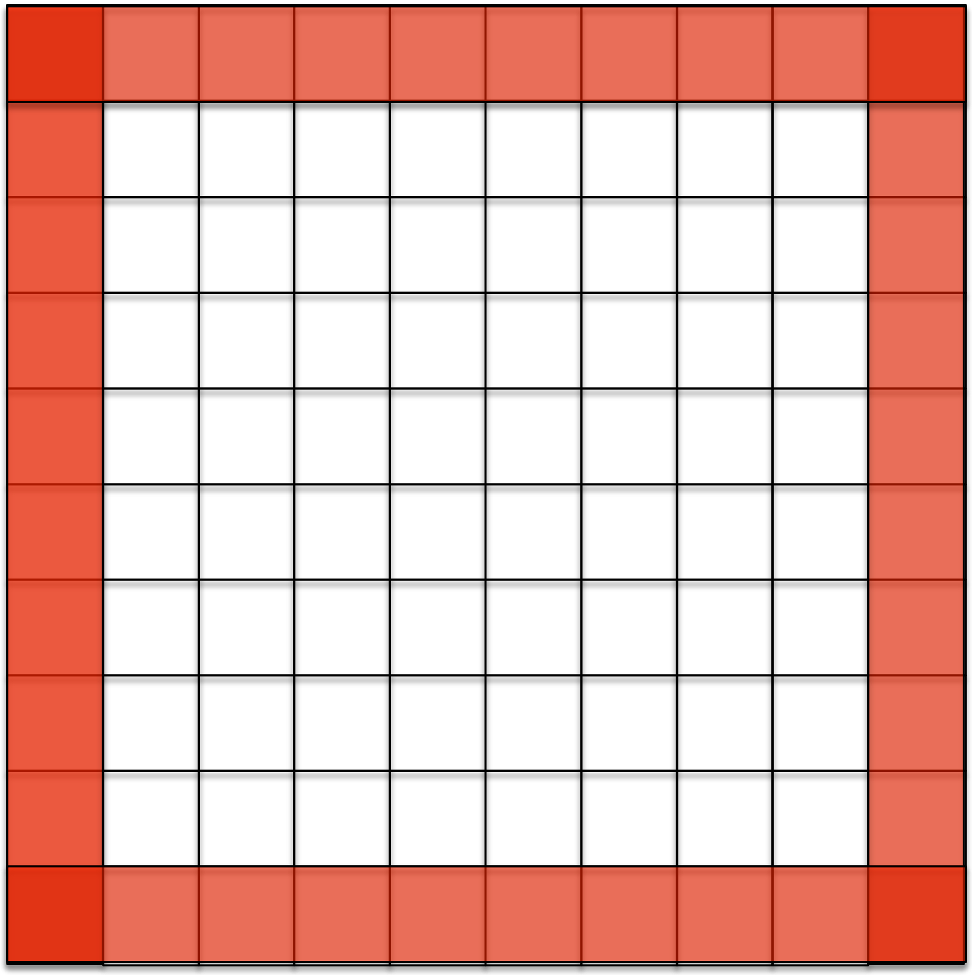
B.

The Border Problem, from

Connecting Mathematical Ideas by

Jo Boaler and Cathy Humphreys

Here’s a 10 x 10 grid.



How many border squares are colored in?

What follows are five central questions to reflect on with regard to mathematics instruction.

To put things simply, classrooms that do well on these will produce students who are mathematically powerful – and classrooms that don't , won't.

Key Questions for Math Classes:

- What were the big ideas, and how did they get developed? Mathematical focus and coherence.
- Did students engage in "productive struggle," or was the math dumbed down to the point where they didn't? Cognitive Demand
- Who had the opportunity to engage? A select few, or everyone? Equity
- Who had a voice? Did students get to say things, develop ownership? Discourse, Student Agency
- Did instruction find out what students know, and build on it? Formative Assessment

Expand each question, with a scoring rubric…

What were the big ideas, and how did they get developed?

Level	**Mathematical Focus and Coherence**
1	Classroom activities are disconnected or unfocused, OR consequential mistakes are left unaddressed.
2	The mathematics discussed is relatively clear and correct, BUT connections between procedures, concepts and contexts (where appropriate) are either cursory or lacking.
3	The mathematics discussed is relatively clear and correct, AND connections between procedures, concepts and contexts (where appropriate) are addressed and explained.
Level	**Mathematical Focus and Coherence**

Did students engage in "productive struggle," or was the math dumbed down to the point where they didn't?

Level	**Cognitive Demand**
1	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.
2	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.
3	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.
Level	**Cognitive Demand**

Who had the opportunity to engage with the math? A select few, or everyone?

Level	**Access to Mathematical Content**
1	There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue. (Such efforts may include expansive framing, use of multiple approaches or representations, or specific attempts at enfranchising disengaged students).
2	There is uneven access or participation but the teacher makes some efforts (which may include expansive framing or use of multiple approaches or representations) to provide mathematical access to a wide range of students.
3	The teacher actively supports (and to some degree achieves) broad and meaningful mathematical participation, **OR** what appear to be established participation structures result in such engagement.
Level	**Access to Mathematical Content**

Who had a voice? Did students get to say things, develop ownership?

Level	**Agency: Authority and Accountability**
1	The teacher initiates conversations. Students' speech turns are short (one sentence or less) and shaped or constrained by what the teacher says or does.
2	Students have a chance to say or explain things, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.
3	Students put forth and defend their ideas. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each others' ideas.
	Agency: Authority and Accountability

Did instruction find out what students know, and build on it?

Level	Uses of Assessment
1	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.
Level	Uses of Assessment

Put everything together:

Level	Mathematical Focus, Coherence and Accuracy	Cognitive Demand	Access	Agency: Authority and Accountability	Uses of Assessment
1	Classroom activities are purely rote, **OR** disconnected or unfocused, **OR** consequential mistakes are left unaddressed.	Classroom activities are structured so that students mostly apply familiar procedures or memorized facts.	Classroom management is problematic to the point where the lesson is disrupted, **OR** a significant number of students appear disengaged and there are no overt mechanisms to support engagement.	The teacher initiates conversations. Students' speech turns are short (one sentence or less) and shaped or constrained by what the teacher says or does.	The teacher may note student answers or work, but student reasoning is not surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	The mathematics discussed is relatively clear and correct, **BUT** connections between procedures, concepts and contexts (where appropriate) are either cursory or lacking.	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges and mostly limit students to providing short responses to teacher prompts.	The class is engaged in mathematical activity, but there is uneven participation and the teacher does not provide structured support for many students to participate in meaningful ways.	Students have a chance to talk about the mathematical content, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific student ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	The mathematics discussed is relatively clear and correct, **AND** connections between procedures, concepts and contexts (where appropriate) are addressed and explained.	The teacher's hints or scaffolds support students in "productive struggle" in building understandings and engaging in mathematical practices.	The teacher actively supports (and to some degree achieves) broad and meaningful participation, **OR** what appear to be established participation structures result in such participation.	Students put forth and defend their ideas. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each others' ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

and you have the dimensions of a framework for assessing lesson quality.

We call this the
“**T**eaching for **R**obust **U**nderstanding of **Math**ematics”
or
TRU Math
scheme.

5.
A tool: Formative Assessment Lessons

The purpose of formative assessments is not simply to show what students "know and can do" after instruction, **but to reveal their current understandings so you can help them improve.**

Important Background Issues

1. Formative assessment is *not* summative assessment given frequently!
2. Scoring formative assessments rather than or in addition to giving feedback destroys their utility (Black & Wiliam, 1998: "inside the black box")
3. This is HARD to do. Tools help!

A tool:
The Formative Assessment Lesson, or FAL, consists of:

A rich "diagnostic" situation

and

Things to do when you see the results of the diagnosis.

A Challenge:

We know that students have many graphing misconceptions, e.g., confusing a picture of a story with a graph of the story in a distance-time graph.

Here’s one way to address the challenge.

CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

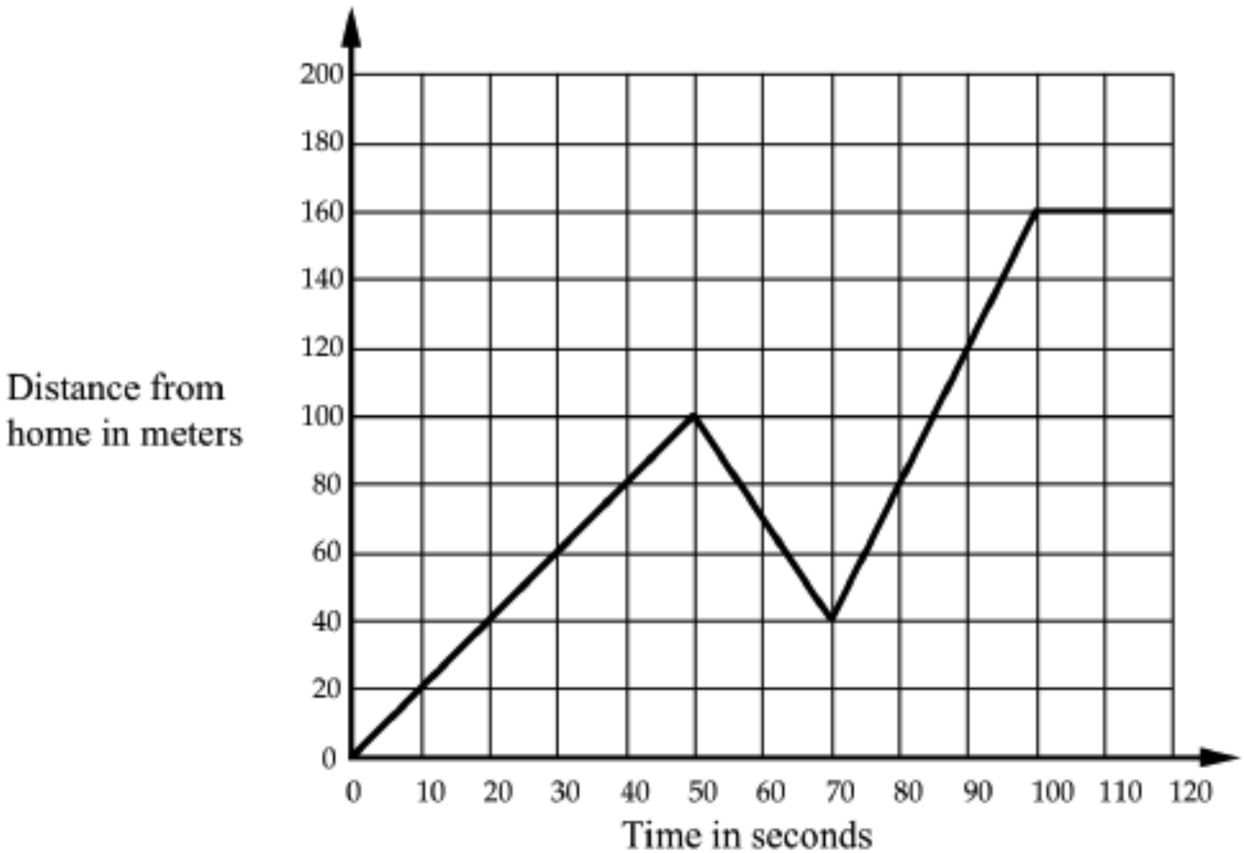
A Formative Assessment Lesson

Interpreting Distance-Time Graphs

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

Before the lesson devoted to this topic, we give a diagnostic problem as homework:

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



Describe what may have happened. Is the graph realistic? Explain.

We point to typical student misconceptions and offer suggestions about how to address them…

Common issues	Suggested questions and prompts
Graph interpreted as a picture E.g. The student assumes that as the graph goes up and down, that Tom's path is going up and down. E.g. The student assumes that a straight line on a graph means that the motion is along a straight path. E.g. The student thinks the negative gradient means Tom has taken a detour.	• *If a person walked in a circle around their home, what would the graph look like?* • *If a person walked at a steady speed up and down a hill, directly away from home, what would the graph look like?* • *In each section of his journey, is Tom's speed steady or is it changing? How do you know?* • *How can you work out Tom's speed in each section of the journey?*
Graph interpreted as speed v time The student has interpreted a positive gradient as speeding up and a negative gradient as slowing down.	• *If a person walked for a mile at a steady speed, away from home, then turned round and walked back home at the same steady speed, what would the graph look like?* • *How does the distance change during the second section of Tom's journey? What does this mean?* • *How does the distance change during the last section of Tom's journey? What does this mean?* • *How can you tell if Tom is travelling away from or towards home?*

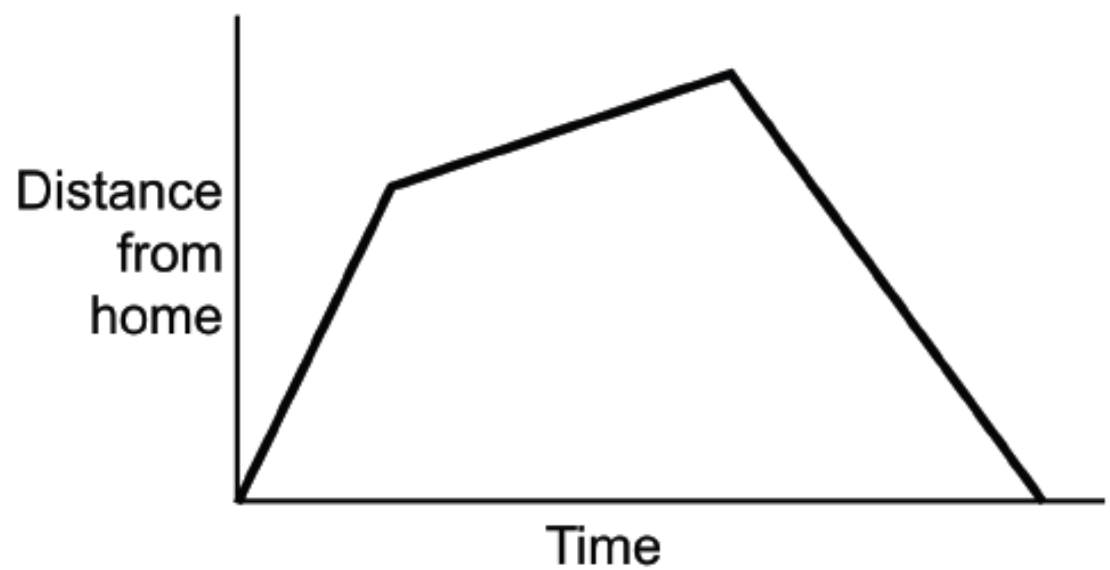
The lesson itself begins with a diagnostic task…

Matching a Graph to a Story

A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.

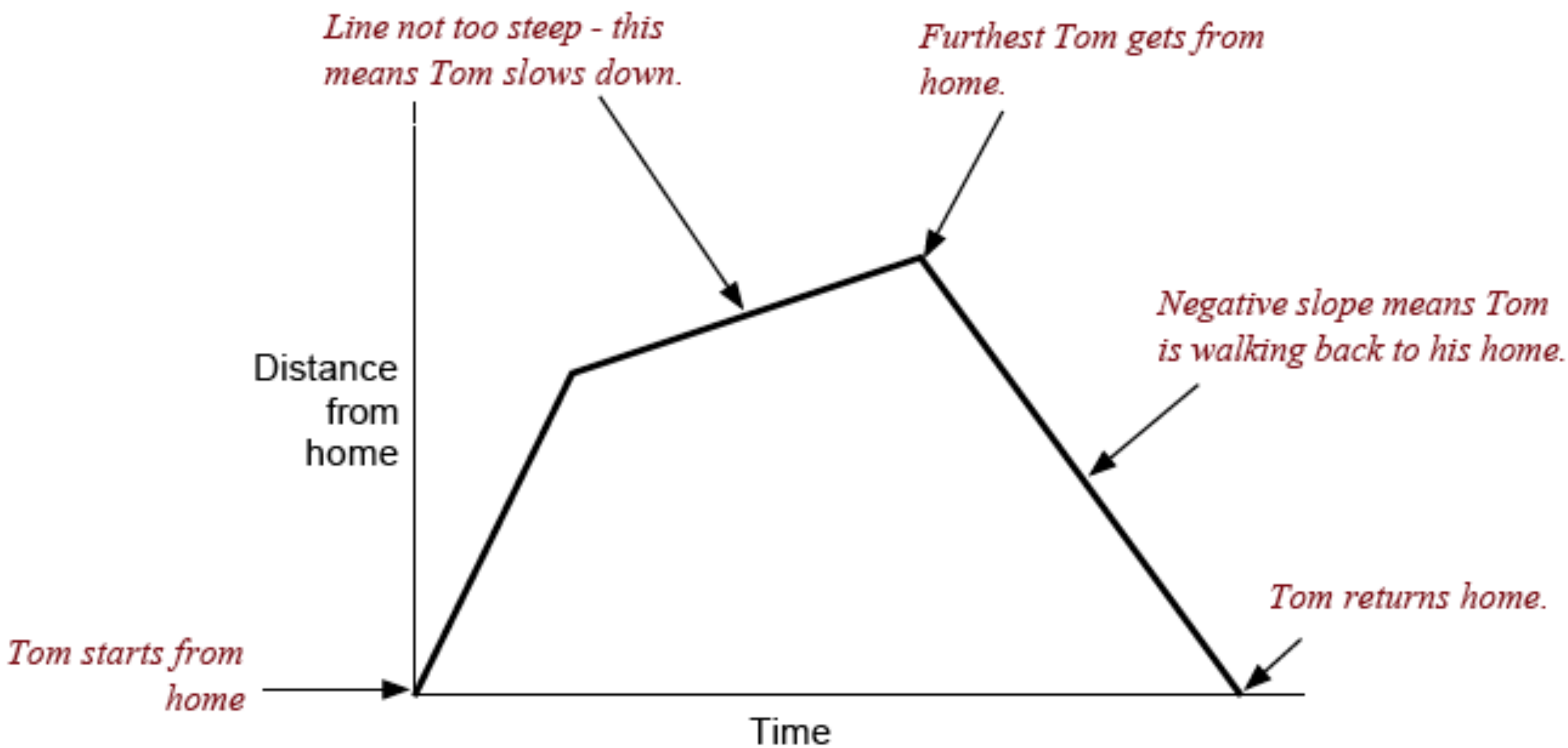
B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.

C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.



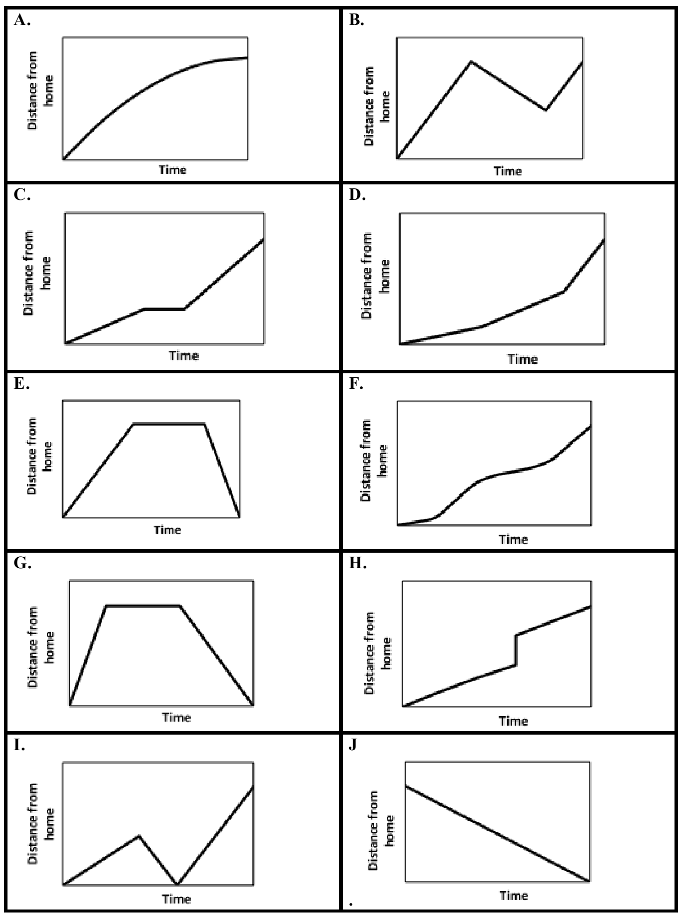
Students are given the chance to annotate and explain…

A graph may end up looking like this:



Follow-up Task: Card Sort
The students make posters.

Card Set A: Distance-Time Graphs



Card Set B: Interpretations

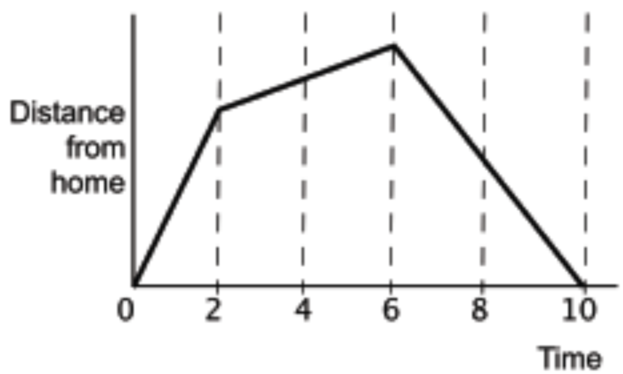
1. Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.	2. Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top and then ran quickly down the other side.
3. Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.	4. Tom walked slowly along the road, stopped to look at his watch, realized he was late, then started running.
5 Tom left his home for a run, but he was unfit and gradually came to a stop!	6. Tom walked to the store at the end of his street, bought a newspaper, then ran all the way back.
7. Tom went out for a walk with some friends when he suddenly realised he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.	8. This graph is just plain wrong. How can Tom be in two places at once?
9. After the party, Tom walked slowly all the way home.	10. Make up your own story!

Students work on converting graphs to tables:

Whole-class discussion: Interpreting tables (15 minutes)

Bring the class together and give each student a mini-whiteboard, a pen, and an eraser. Display Slide 5 of the projector resource:

Making Up Data for a Graph



Time	Distance
0	
2	
4	
6	
8	
10	

On your whiteboard, create a table that shows possible times and distances for Tom's journey.

Tables are added to the card sort…

Card Set C: Tables of data

A.

Time	Distance
0	0
1	40
2	40
3	40
4	20
5	0

B.

Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120

C.

Time	Distance
0	0
1	18
2	35
3	50
3	85
5	120

D.

Time	Distance
0	0
1	40
2	80
3	60
4	40
5	80

E.

Time	Distance
0	0
1	20
2	40
3	40
4	40
5	0

F.

Time	Distance
0	0
1	30
2	60
3	0
4	60
5	120

G.

Time	Distance
0	0
1	20
2	40
3	40
4	80
5	120

H.

Time	Distance
0	0
1	45
2	80
3	105
4	120
5	125

I

Time	Distance
0	120
1	96
2	72
3	48
4	24
5	0

J. Make this one up!

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

K.

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

And the class compares solutions together.

Here' s another FAL:

Evaluating Statements About Length and Area

Mathematical goals

This lesson unit is intended to help you assess how well students can:

- Understand the concepts of length and area.
- Use the concept of area in proving why two areas are or are not equal.
- Construct their own examples and counterexamples to help justify or refute conjectures.

Common Core State Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

G-CO Prove geometric theorems.

This lesson involves a range of *mathematical practices*, with emphasis on:

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

Shape Statement 1

1. James says:

> If you draw two shapes, the shape with the greater area will also have the longer perimeter.

Is James' statement Always, Sometimes or Never True?

Fully explain and illustrate your answer.

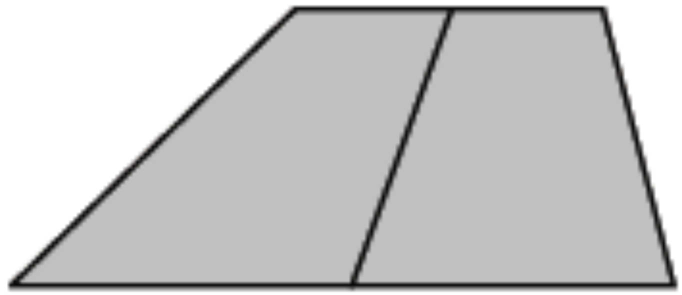
Shape Statement 2

2. Clara says:

> If you join the midpoints of the opposite sides of a trapezoid, you split the trapezoid into two equal areas.

Is Clara's statement Always, Sometimes or Never True?

Fully explain and illustrate your answer.



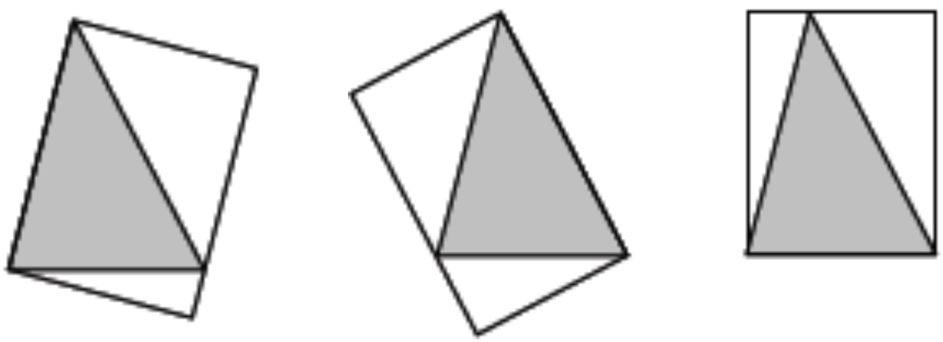
Shape Statement 3

3. Alex says:

There are three different ways of drawing a rectangle around a triangle, so that it passes through all three vertices and shares an edge. The areas of the rectangles are equal.

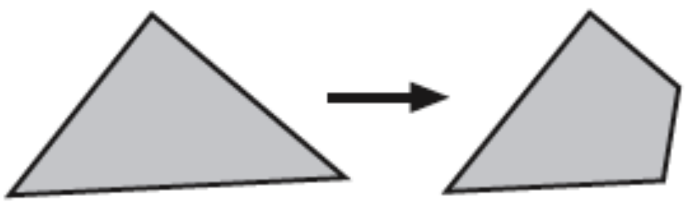
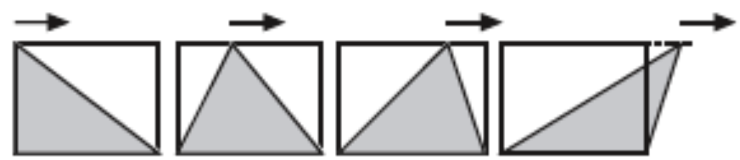
Is Alex's statement Always, Sometimes or Never True?

Fully explain and illustrate your answer.



There are more great tasks, e.g.,

Always, Sometimes, or Never True?

A **Cutting Shapes**	B **Sliding a Triangle**
	
When you cut a piece off a shape you: (a) Reduce its area. (b) Reduce its perimeter.	If you slide the top corner of a triangle from left to right: (a) Its area stays the same. (b) Its perimeter changes.

And, the students develop critiquing skills. The task:

Diagonals of a Quadrilateral

If you draw in the two diagonals of a quadrilateral, you divide the quadrilateral into four equal areas.

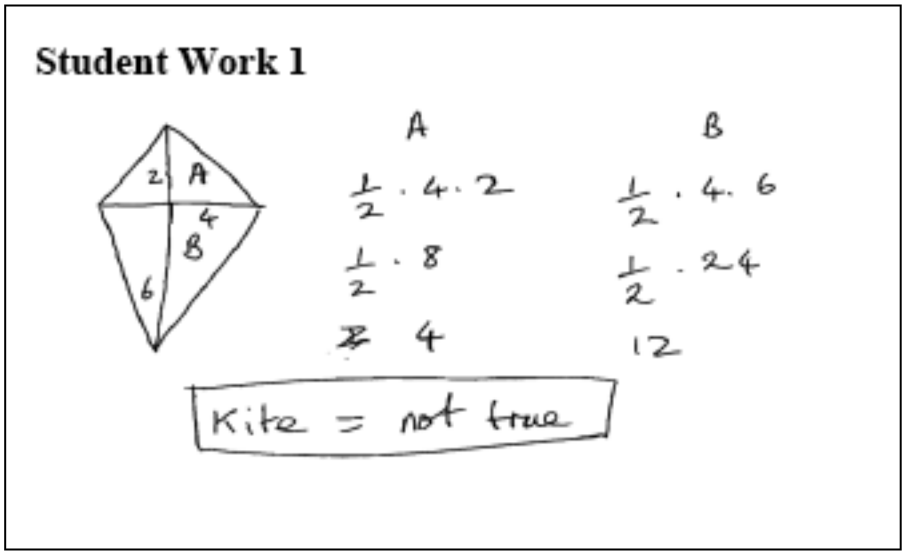
Is this statement always, sometimes or never true?

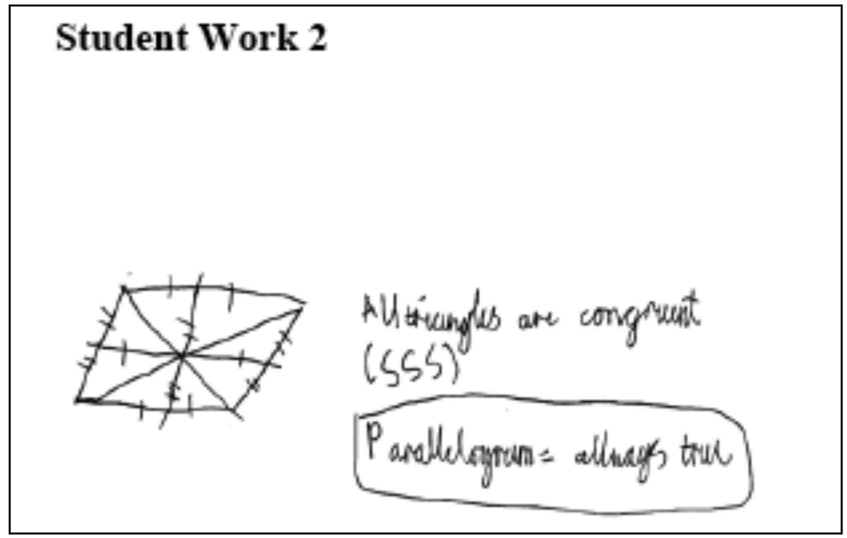
If you think the statement is always true or never true, then how would you convince someone else?

If you think the statement is sometimes true, would you be able to identify all the cases of a quadrilateral where it is true/not true?

They discuss the task, and sort out the mathematics. Then…

They' re given other (hypothetical) students' work…





And helped to critique it.

These are central skills called for in CCSSM.

PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

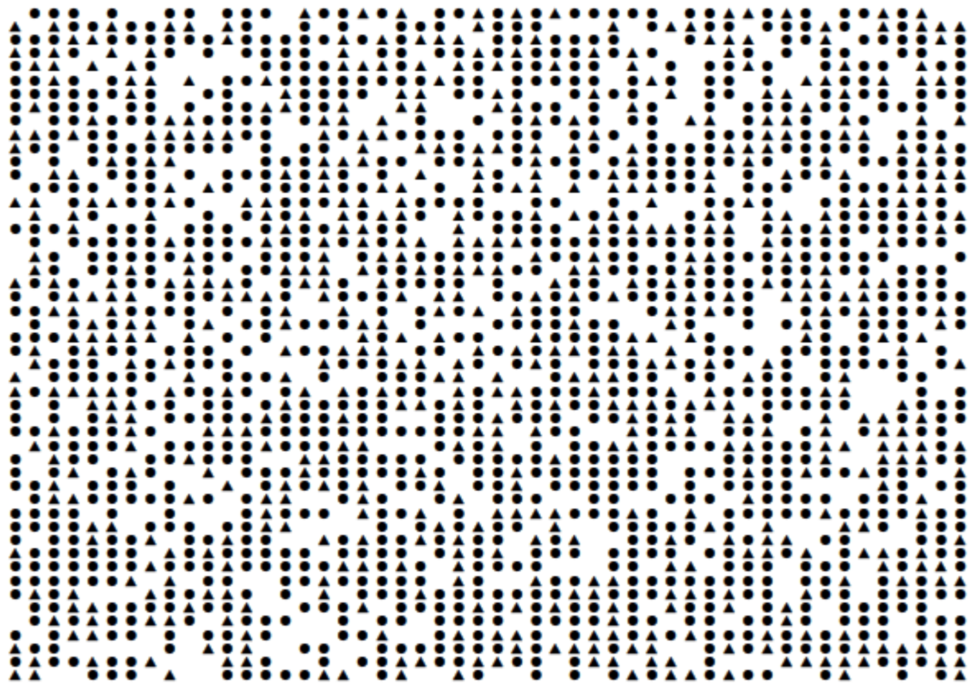
A Formative Assessment Lesson

Estimating: *Counting Trees*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

Counting Trees

The diagram shows some trees in a tree farm.



The circles ● show old trees and the triangles ▲ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one by one.

- Think of a method you could use to estimate the number of trees of each type.
- Explain the method fully.
- Use your method to estimate the number of old trees and young trees.0

The Mathematics Assessment Project’s goals are to:

- Help students grapple with core content and practices in CCSSM, and prepare them for the rich assessments they should experience;
- Support formative assessment; and
- Do so in “curriculum-embeddable” ways.

We're building 20 FALs at each grade from 6 through 10.

They're **FREE**, at http://map.mathshell.org/materials.

Just google

Mathematics Assessment Project

6.
A tool: The Question Version of TRU Math, or Q-TRU

These are the dimensions of productive classrooms, and thus the things you have to work on in professional development:

1. Rich Mathematics
2. Cognitive Demand
3. Equitable Opportunities to Engage
4. Opportunities to talk mathematics
5. Building on Student thinking

Think about sets of questions for each dimension that stimulate conversation and reflection…

- Pre-lesson
- After the lesson
- When planning next steps

Start with the core questions:

- How do mathematical ideas from this unit/ course develop in this lesson/lesson sequence?
- What opportunities do students have to make their own sense of mathematical ideas?
- Who does and doesn't participate in the mathematical work of the class, and how?
- What opportunities do students have to explain their own and respond to each other's mathematical ideas?
- What do we know about each student's current mathematical thinking, and how can we build on it?

And think about expanding them:

1. MATHEMATICAL FOCUS AND COHERENCE		
Pre-observation	**Reflecting After a Lesson**	**Planning Next Steps**
How will important mathematical ideas develop in this lesson and unit?	How did students actually engage with important mathematical ideas in this lesson?	How can we connect the mathematical ideas that surfaced in this lesson to future lessons?

Think about:

- What the mathematical goals for the lesson are.
- What connections exist among important ideas in this lesson and important ideas in past and future lessons.
- How math procedures in the lesson are justified and connected with important ideas.
- How we see/hear students actually engage with mathematical ideas during class.
- Which students get to engage deeply with important mathematical ideas.
- How future instruction could create opportunities for more studentxs to engage more deeply with mathematical ideas.

2. COGNITIVE DEMAND

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities will students have to make their own sense of important mathematical ideas?	What opportunities did students have to make their own sense of important mathematical ideas?	How can we create more opportunities for students to make their own sense of important mathematical ideas?

Think about:

- What opportunities exist for students to struggle with mathematical ideas.
- How students' struggles may support their engagement with mathematical ideas.
- How the teacher responds to students' struggles and how these responses support students to engage meaningfully.
- What resources (other students, the teacher, notes, texts, technology, manipulatives, various representations, etc.) are available for students to use when they encounter struggles.
- What resources students actually use and how they might be supported to make better use of resources.
- Which students get to engage deeply with important mathematical ideas.
- How future instruction could create opportunities for more students to engage more deeply with mathematical ideas.
- What community norms seem to be evolving around the value of struggle and mistakes.

3. ACCESS		
Pre-observation	**Reflecting After a Lesson**	**Planning Next Steps**
What opportunities exist for each student to participate in the mathematical work of the class?	Who did and didn't participate in the mathematical work of the class, and how?	How can we create opportunities for each student to participate in the mathematical work of the class?

Think about:

- What range of ways students can and do participate in the mathematical work of the class (talking, writing, leaning in, listening hard; manipulating symbols, making diagrams, interpreting graphs, using manipulatives, connecting different strategies, etc.).
- Which students participate in which ways.
- Which students are most active when, and how we can create opportunities for more students to participate more actively.
- What opportunities various students have to make meaningful mathematical contributions.
- Access to and development of students' academic language.
- How norms (or interactions, or lesson structures, or task structure, or particular representations, etc.) facilitate or inhibit participation for particular students.
- What teacher moves might expand students' access to meaningful participation (such as modeling ways to participate, providing opportunities for practice, holding students accountable, pointing out students' successful participation).
- How to support particular students we are concerned about (in relation to learning, issues of safety, participation, etc.).

4. AGENCY AND AUTHORITY

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities exist in the lesson for students to explain their own and respond to each other's mathematical ideas?	What opportunities did students have to explain their own and respond to each other's mathematical ideas?	What opportunities can we create in future lessons for more students to explain their own and respond to each other's mathematical ideas?

Think about:

- Who generates the mathematical ideas that get discussed.
- Who evaluates and/or responds to others' ideas.
- How deeply students get to explain their ideas.
- How the teacher responds to student ideas (evaluating, questioning, probing, soliciting responses from other students, etc.).
- How norms around students' and teachers' roles in generating mathematical ideas are developing.
- How norms around what counts as mathematics (justifying, experimenting, practicing, etc.) are developing.
- Which students get to explain their own and respond to others' ideas in a meaningful way.

5. USES OF ASSESSMENT

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What do we know about each student’s current mathematical thinking, and how does this lesson build on it?	What did we learn in this lesson about each student’s mathematical thinking? How was this thinking built on?	Based on what we learned about each student’s mathematical thinking, how can we (1) learn more about it and (2) build on it?

Think about:

- What opportunities exist for students to share their mathematical ideas and reasoning.
- Which different ways exist for students to share their mathematical ideas and reasoning (writing on paper, speaking, writing on the board, creating diagrams, demonstrating with manipulatives, etc.).
- Who students get to share their ideas with.
- How students are likely to make sense of the mathematics in the lesson and what responses might build on that thinking.
- What things we can try (e.g., tasks, lesson structures, questioning prompts such as those in FALs) to surface student thinking, especially the thinking of students whose mathematical ideas we don't know much about yet.
- What we know and don't know about how each student is making sense of the mathematics we are focusing on.
- What opportunities exist to build on students' mathematical thinking, and how teachers and/or other students take up these opportunities.

But these are hard to learn, and take time to work on!

And, they can be sabotaged easily, by classroom observers who (for example) believe “A good classroom is a quiet classroom.”

So, whaddya do?

7.
A Grand Plan for Change

Here’s what we’d do...

For administrators

For teachers

For teacher communities

For administrators:

Teach them using FALs, and debrief on our teaching using Q-TRU.

(If they know what it’s like to be a student in a productive class, they’re less likely to disrupt them.)

For Teachers:

Once a month, teach them a FAL they'll be teaching that month, and debrief on our teaching using Q-TRU.

Have coaches and administrators use Q-TRU for discussing lesson planning and observations.

For Teacher Communities:

Build Lesson Study Communities in which teachers have the opportunity to watch each other teaching the FALs. Support the community in learning to debrief using Q-TRU.

By the end of a year,

The five dimensions should be something that come to mind automatically, as a mechanism for planning and reflection.

But, each dimension is hard, and a year isn't enough.

So take more time. In year 1 we might start with dimensions 1 and 4:

- Rich mathematics
- Opportunities to talk mathematics

And we'd build on those in Year 2.

8.
Discussion