Fluctuations and the QCD Critical Point

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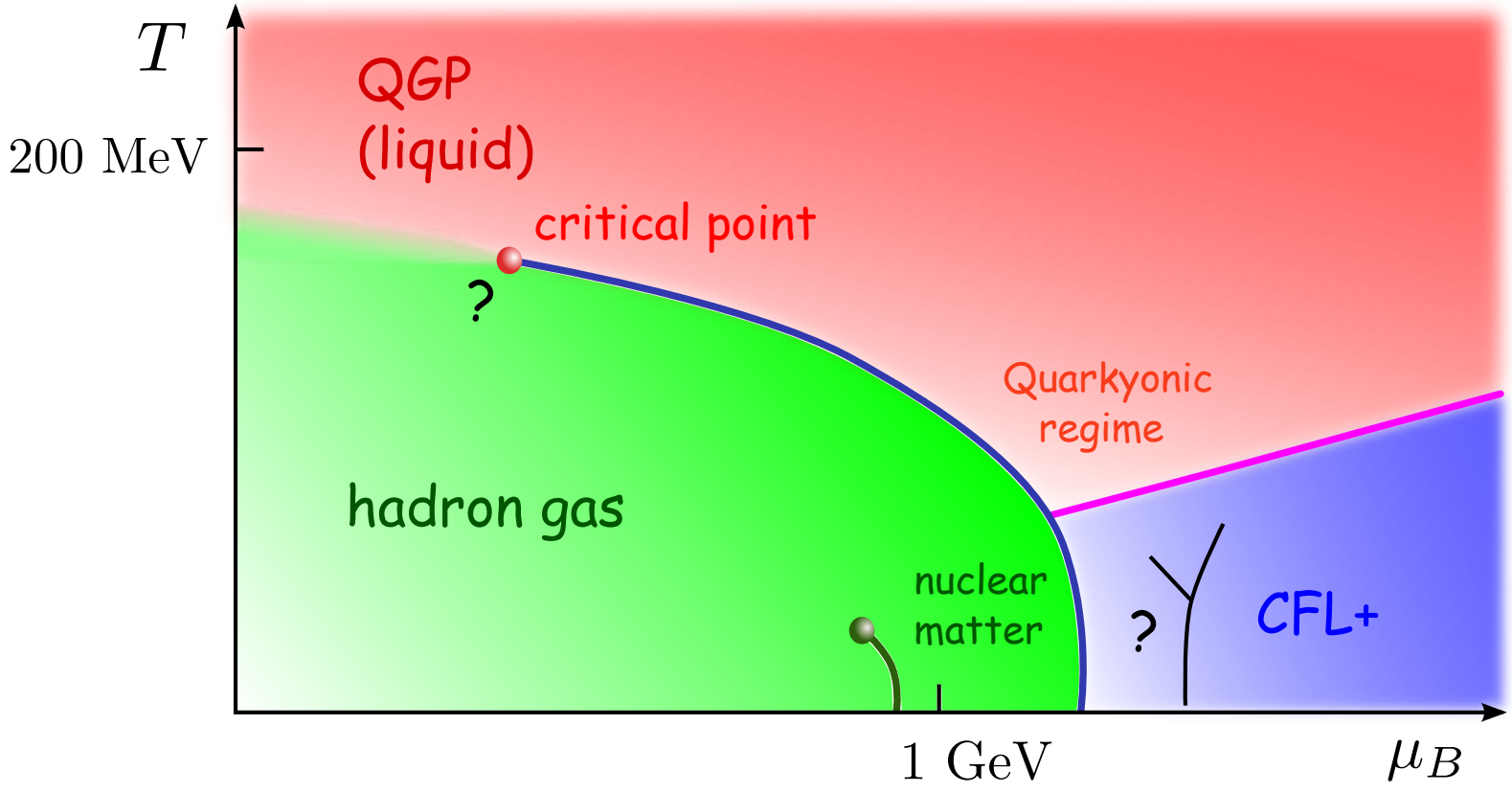
UIC

Outline

1. QCD phase diagram, critical point and fluctuations
   - Critical fluctuations and correlation length
   - Non-gaussian moments and universality
2. Beam energy scan
   - Mapping to QCD and observables
   - Intriguing data from RHIC BES I
   - Acceptance dependence

QCD Phase Diagram *(a theorist's view)*



Lattice at $\mu_B \lesssim 2T$

Critical point – a fundamental feature of QCD phase diagram and a major goal for H.I.C.

Why fluctuations are large at a critical point?

- The key equation:

$$P(\sigma) \sim e^{S(\sigma)} \qquad \textit{(Einstein 1910)}$$

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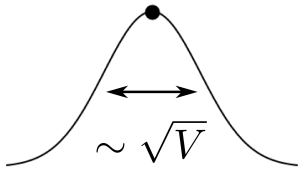
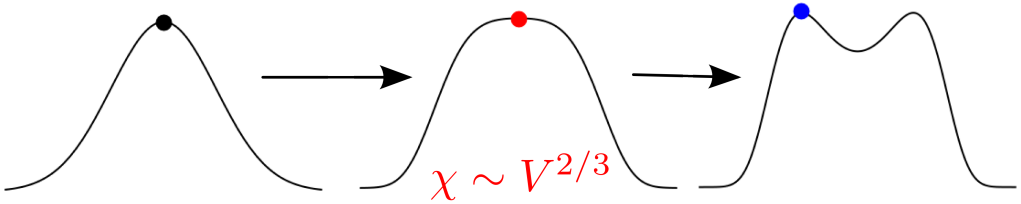


- At the critical point $S(\sigma)$ "flattens". And $\chi \equiv \langle \sigma^2 \rangle / V \to \infty$.



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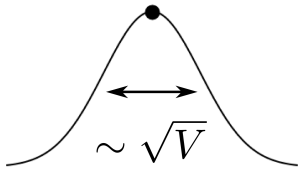

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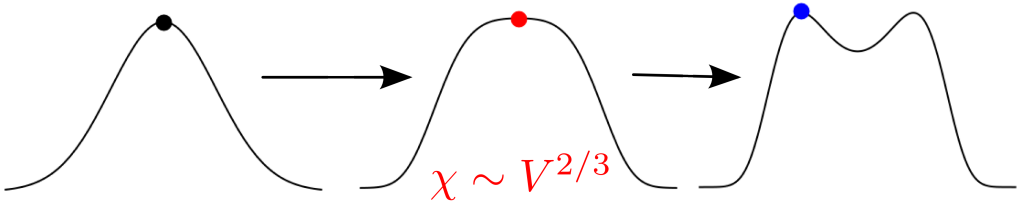

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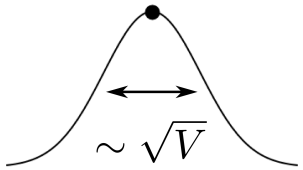

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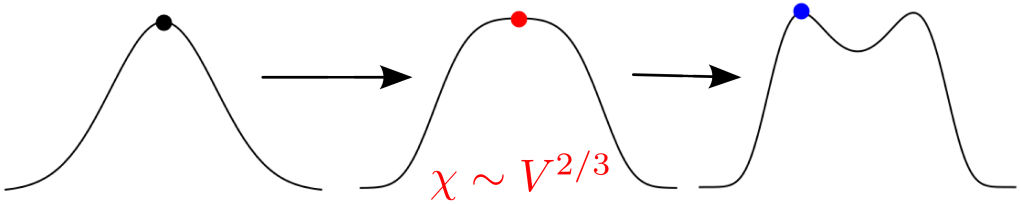

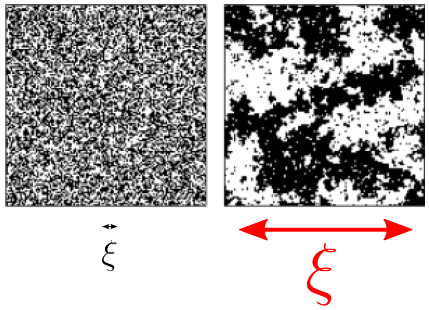

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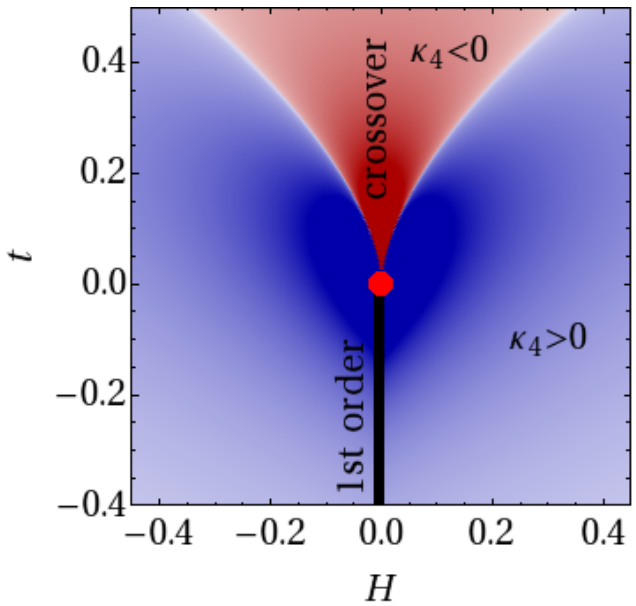
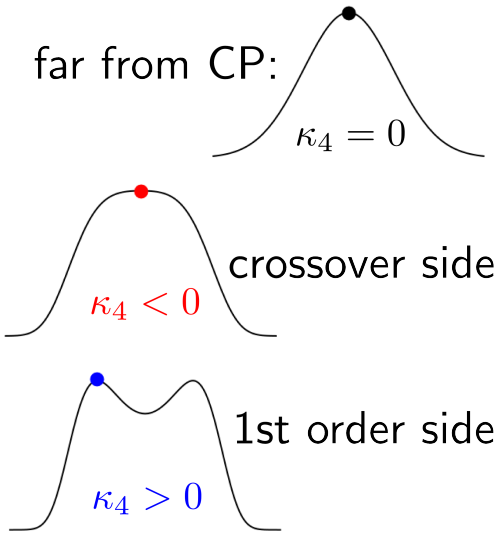
Higher order cumulants

- Higher cumulants (shape of $P(\sigma)$) depend stronger on ξ.

  E.g., $\langle\sigma^2\rangle \sim V\xi^2$ while $\langle\sigma^4\rangle_c \sim V\xi^7$
- Higher moments also depend on which side of the CP we are.

  This dependence is also universal.
- Using Ising model variables:





Why ξ is finite

System expands and is out of equilibrium

In this talk – *equilibrium* fluctuations. The only dynamical effect we consider is the one which makes ξ finite:

Critical slowing down. Universal scaling law:

$$\xi \sim \tau^{1/z}, \qquad \text{where } 1/\tau \text{ is expansion rate}$$

and $z \approx 3$ (Son-MS).

Estimates: $\xi \sim 2 - 3$ fm (Berdnikov-Rajagopal, Asakawa-Nonaka).

- Dynamical description of fluctuations is essential and is work in progress. BEST COLLABORATION

Experiments do not measure σ.

Mapping to QCD and experimental observables

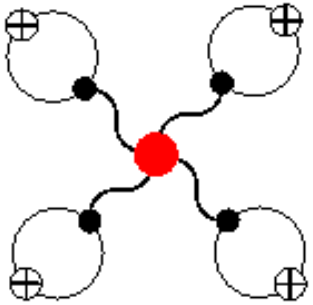
Observed fluctuations are not the same as σ, but related:

Think of a collective mode described by field σ such that $m = m(\sigma)$:

$$\delta n_{\boldsymbol{p}} = \delta n_{\boldsymbol{p}}^{\text{free}} + \frac{\partial \langle n_{\boldsymbol{p}} \rangle}{\partial \sigma} \times \delta\sigma$$

The cumulants of multiplicity $M \equiv \int_{\boldsymbol{p}} n_{\boldsymbol{p}}$:

- $\kappa_4[M] = \underbrace{\langle M \rangle}_{\text{baseline}} + \kappa_4[\sigma] \times g^4 \underbrace{\Big($$ $\Big)^4}_{\sim M^4} + \ldots,$



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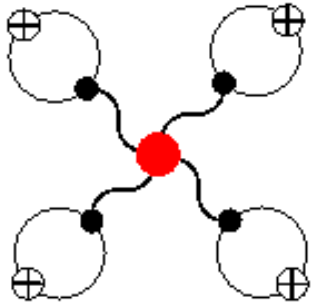

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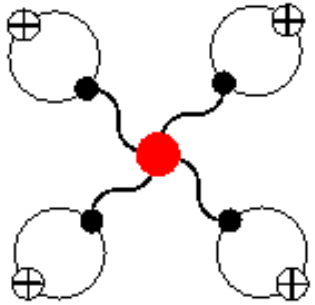

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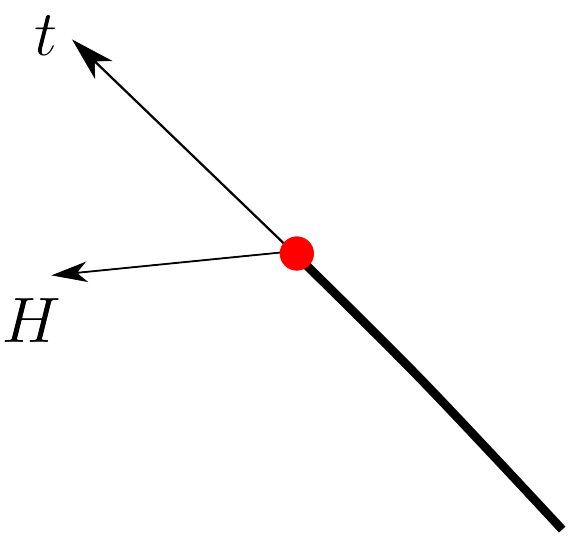
- NB: Sensitivity to M_{accepted}: $(\kappa_4)_\sigma \sim M^4$ (number of 4-tets).

Beam Energy Scan



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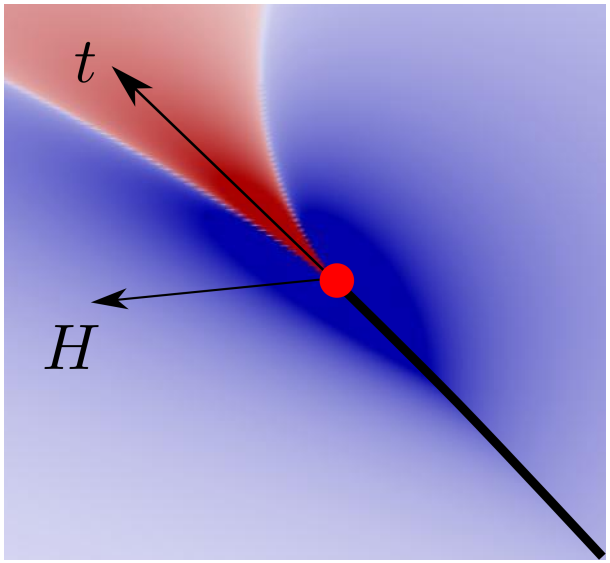

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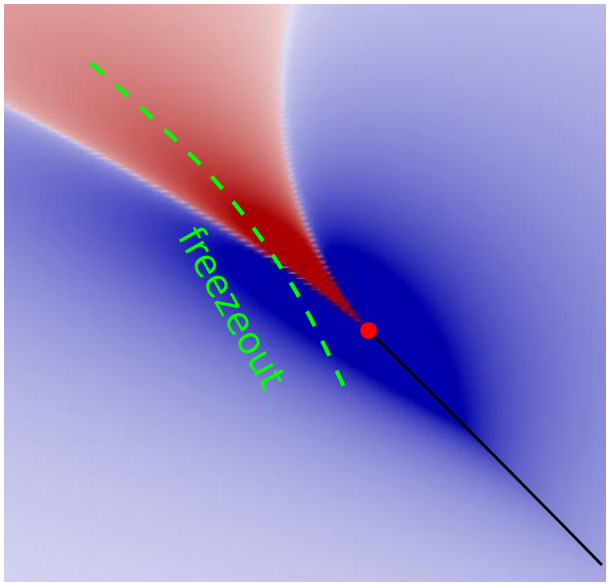

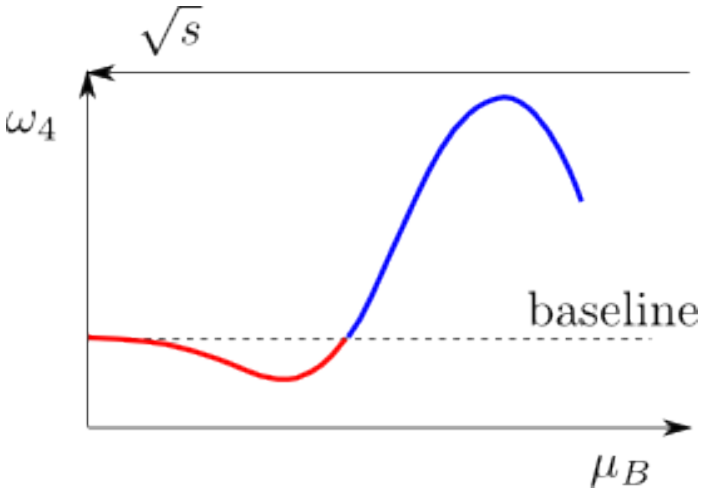

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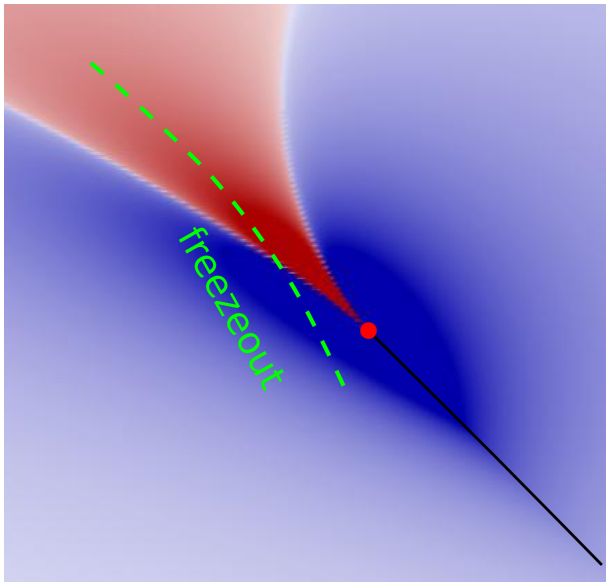

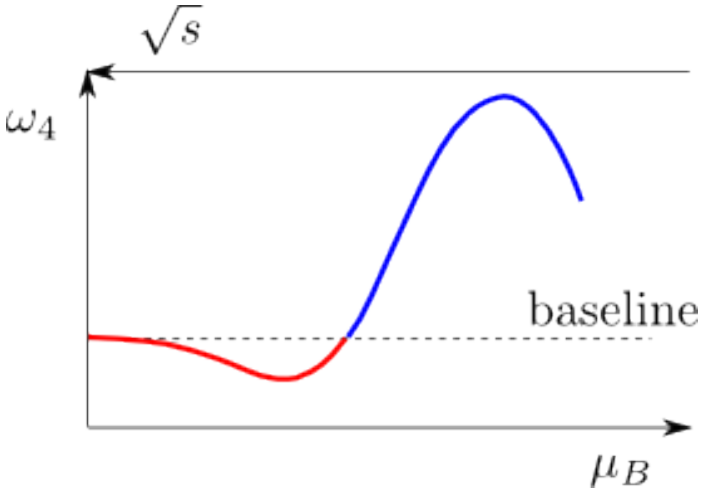

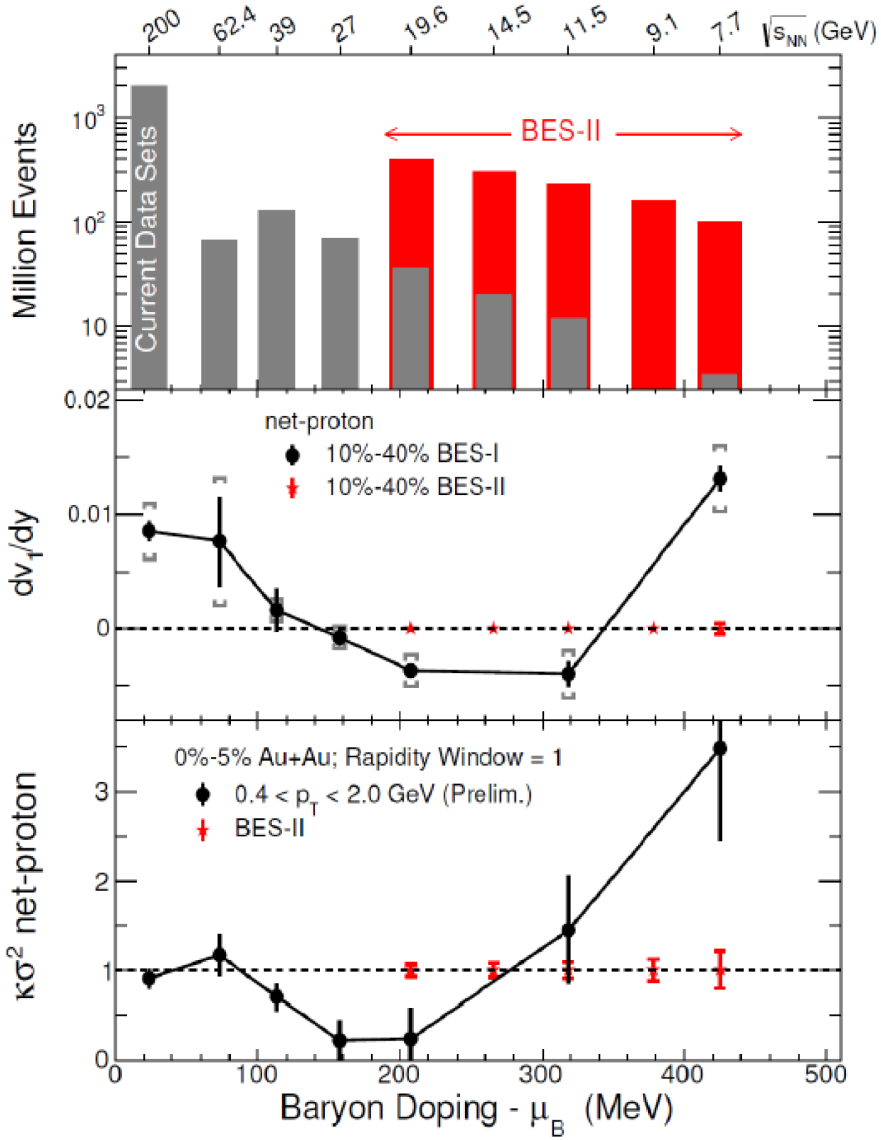

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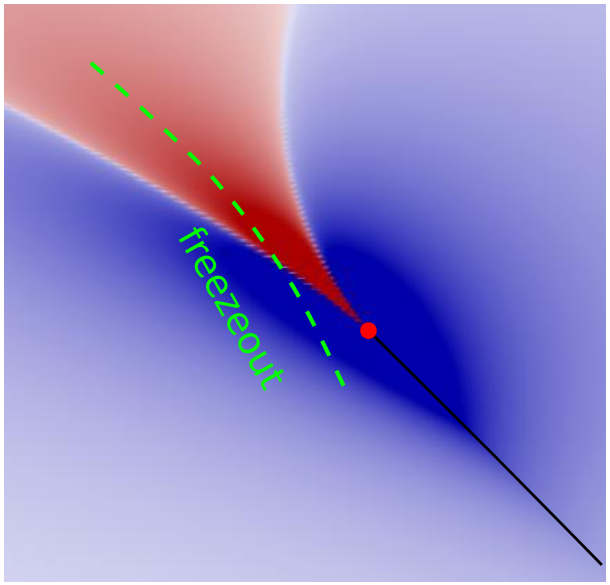

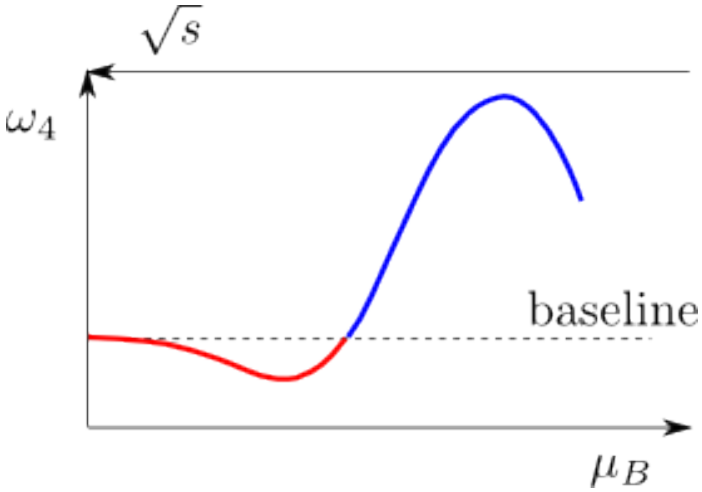

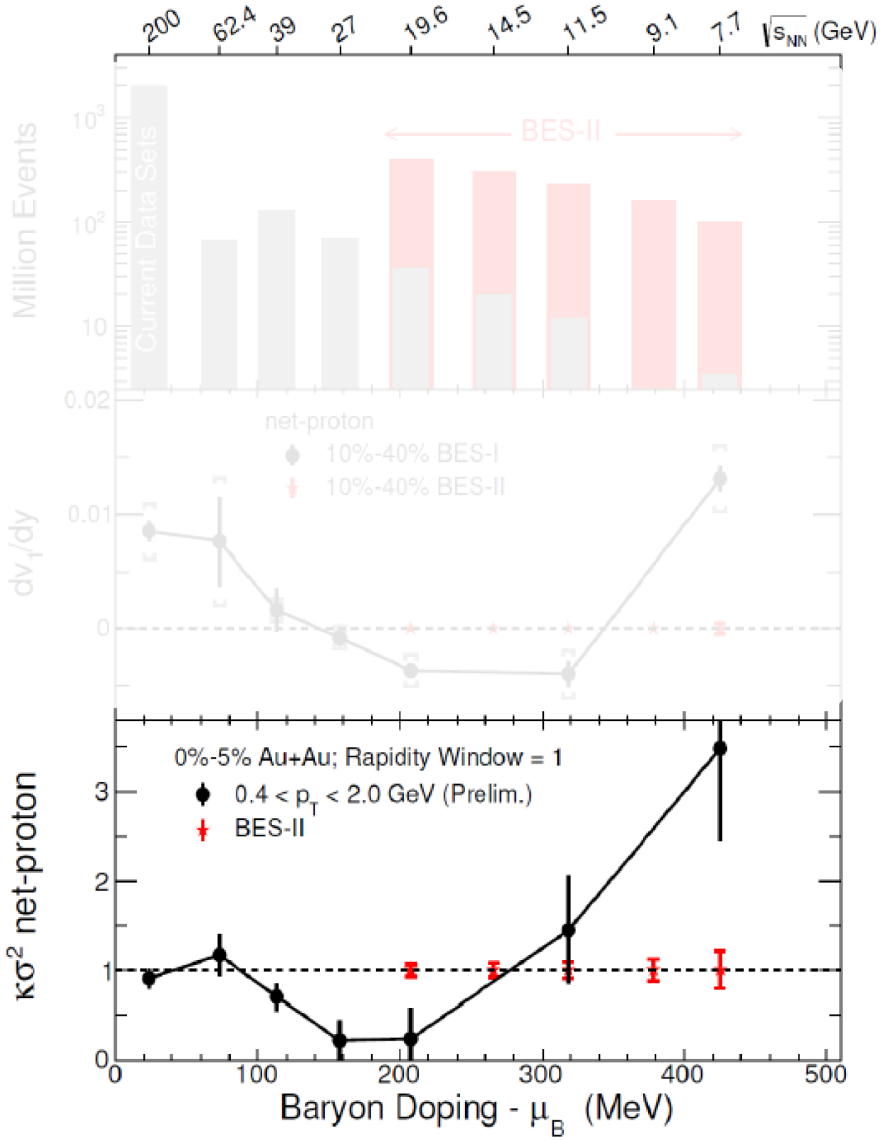

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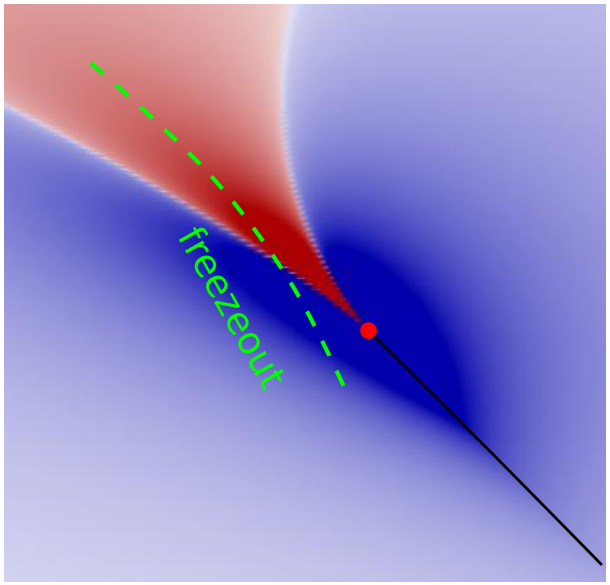
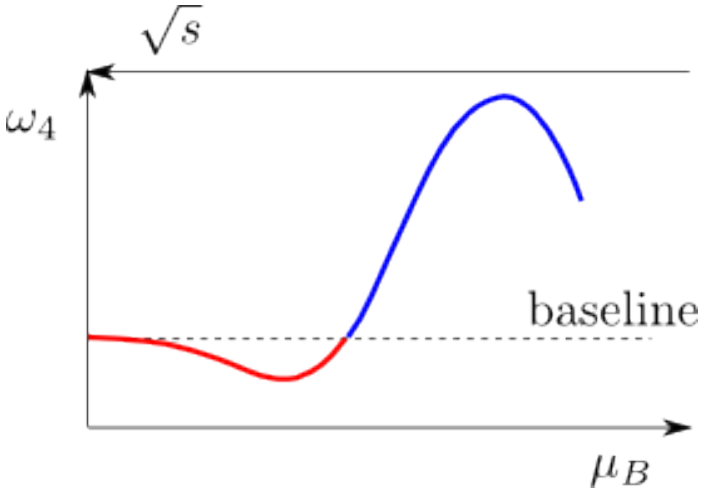
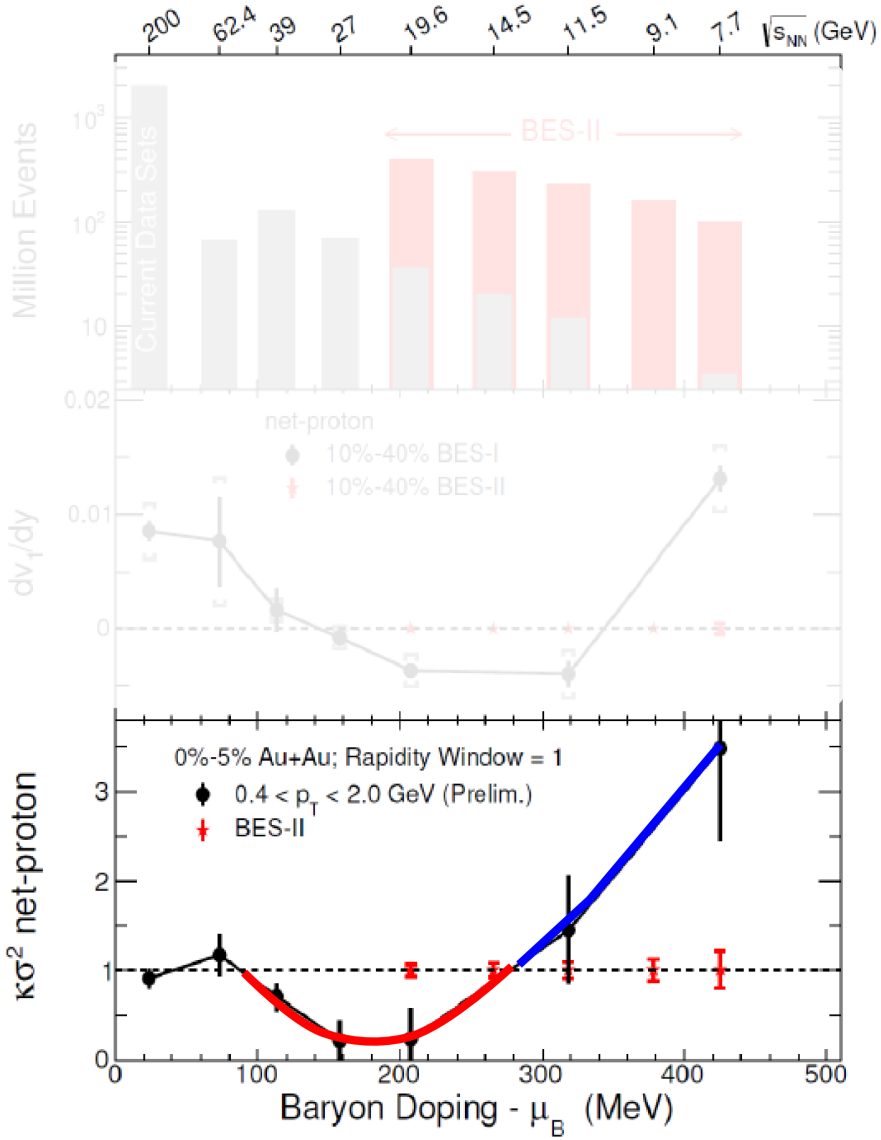


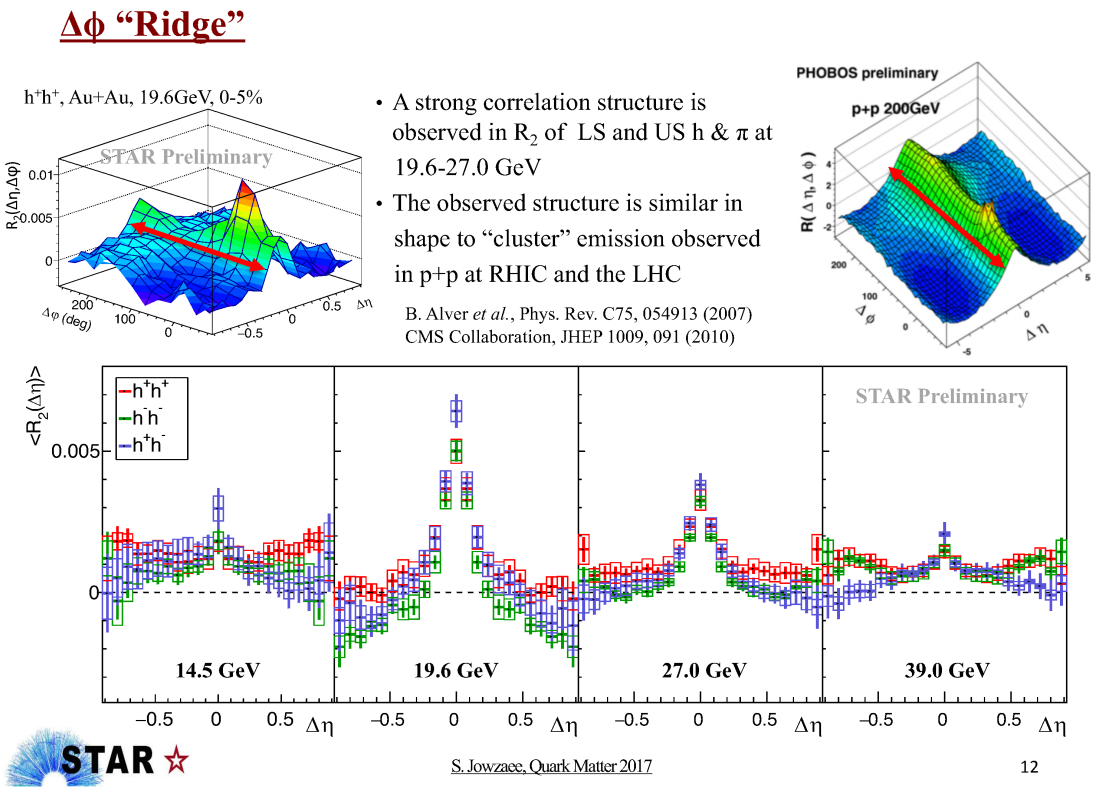




"intriguing hint" (2015 LRPNS)

QM2017 update: another intriguing hint

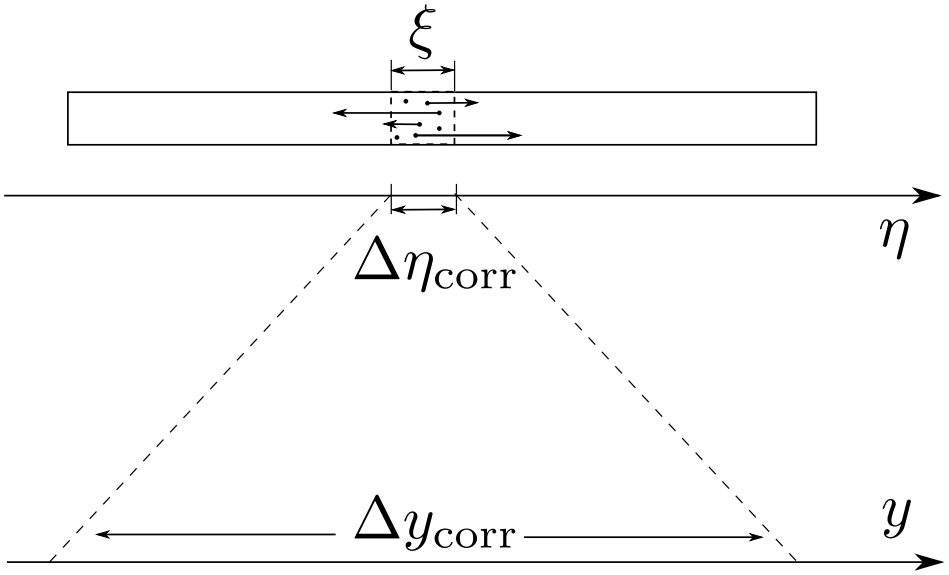
Preliminary, but very interesting:



- Non-monotonous $\sqrt{s}$ dependence with max near 19 GeV.
- Charge/isospin blind.
- $\Delta\phi$ (in)dependence is as expected from critical correlations.
- Width $\Delta\eta$ suggests soft thermal pions – but p_T dependence need to be checked.
- But: no signal in R_2 for K or p.

Acceptance dependence

Correlations – spatial vs kinematic



$\xi \sim 1 - 3$ fm

$$\Delta\eta_{\rm corr} = \frac{\xi}{\tau_{\rm f}} \sim 0.1 - 0.3$$

Particles within $\Delta\eta_{\rm corr}$ have thermal rapidity spread. Thus

$$\Delta y_{\rm corr} \sim 1 \gg \Delta\eta_{\rm corr}$$

Acceptance dependence – two regimes

How do cumulants depend on acceptance?

Let $\kappa_n(M)$ be a cumulant of M – multiplicity of *accepted*, say, protons.

- $\Delta y \gg \Delta y_{\rm corr}$ – CLT applies.

  $\kappa_n \sim M$

  or $\omega_n \equiv \dfrac{\kappa_n}{M} \to {\rm const}$ – an "intensive", or volume indep. measure

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- $\Delta y \ll \Delta y_{\rm corr}$ – more typical in experiment.

  Subtracting trivial (uncorrelated, Poisson) contribution:

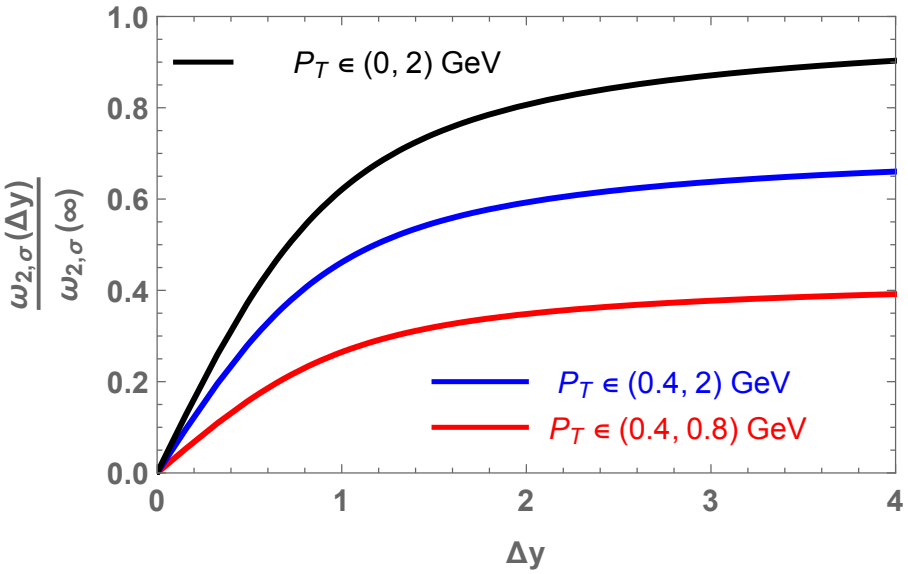
  $\kappa_n - M \sim M^n$ – proportional to number of correlated n-plets;

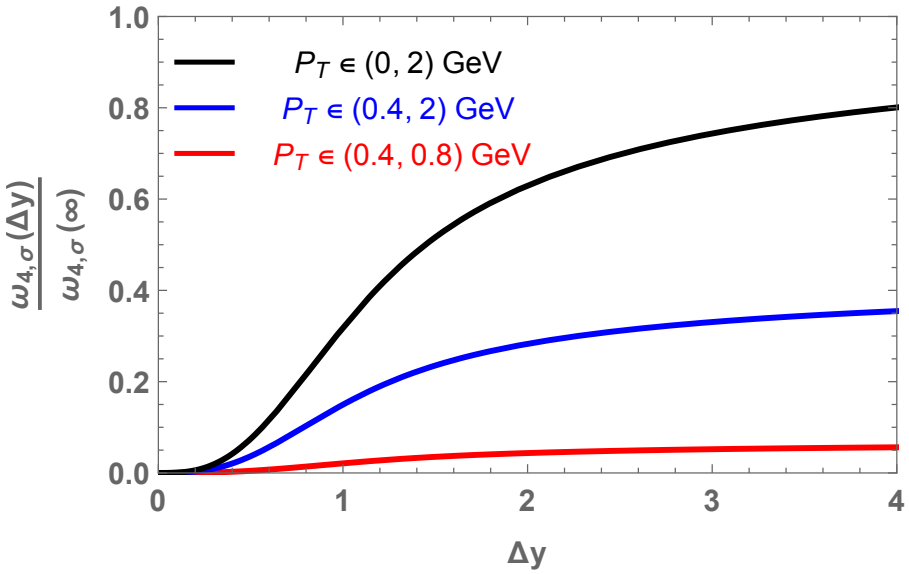
  or $\omega_n - 1 \sim M^{n-1}$.

Critical point fluctuations vs acceptance

Proton multiplicity cumulants ratio at 19.6 GeV: $\omega_{n,\sigma} \equiv \omega_n - 1$

grows as $(\Delta y)^{n-1}$ and saturates at $\Delta y \sim 1 - 2$.





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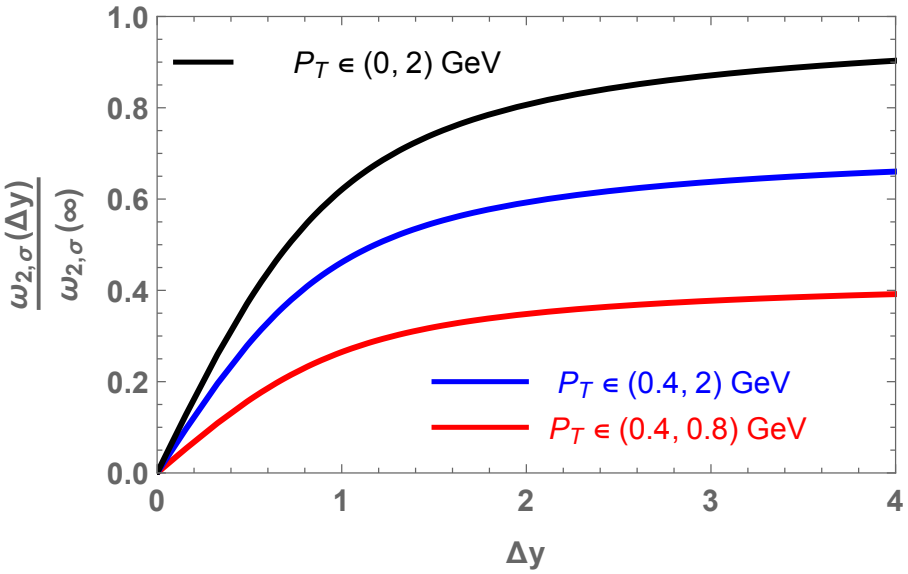
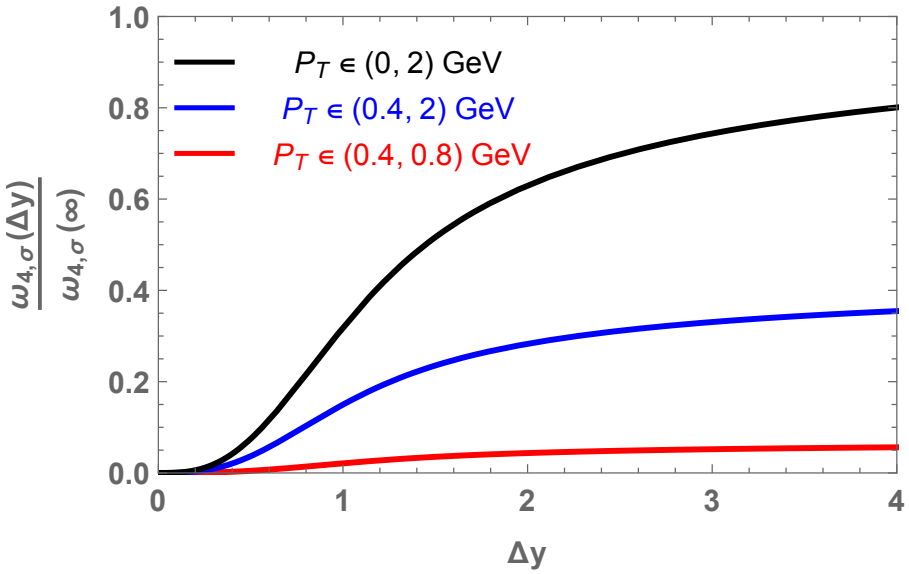
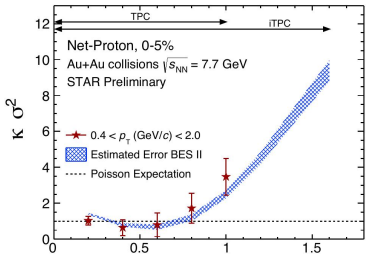




p_T and rapidity cuts have qualitatively similar effects.

- Wider acceptance improves signal/error: errors grow slower than M^n.



Concluding summary

- A fundamental question for Heavy-Ion collision experiments:

  Is there a critical point on the boundary between QGP and hadron gas phases?
- Intriguing data from RHIC BES I.

  Needed: better understanding. More data from BES II.
- Critical fluctuations have many universal properties.
- Characteristic non-monotonic $\sqrt{s}$ dependence of fluctuations (with sign change for non-gaussian moments) – a CP signature.
- Increase of signal with rapidity acceptance is characteristic of critical fluctuations.
- Dynamical description of fluctuations is essential and is work in progress.