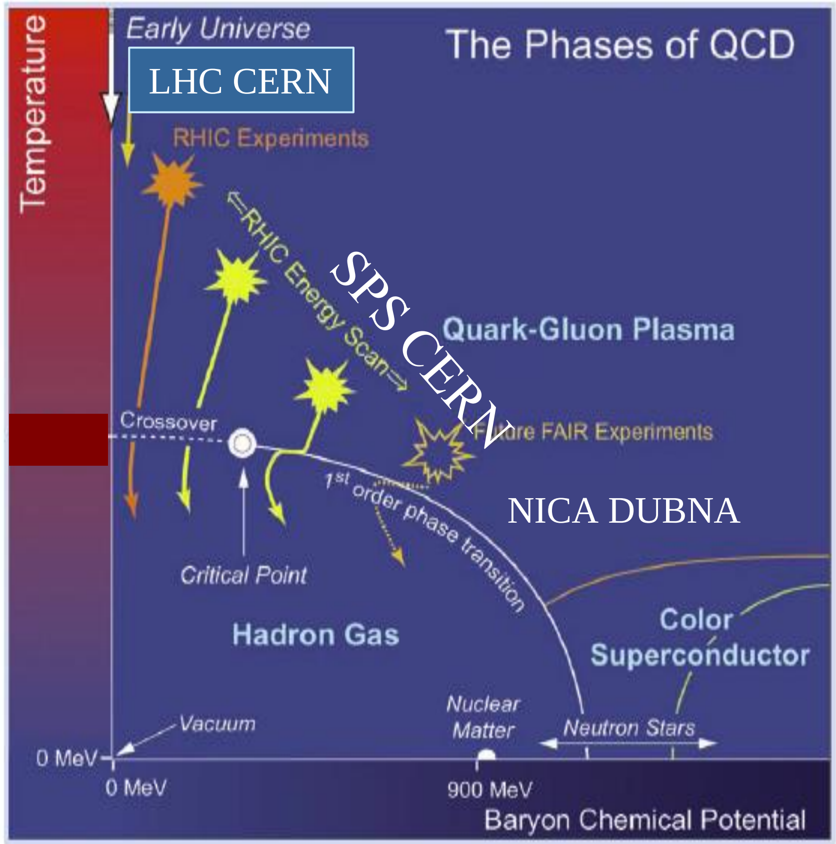
Probing QCD Phase Diagram in Heavy Ion Collisions

Krzysztof Redlich, University of Wroclaw & EMMI/GSI

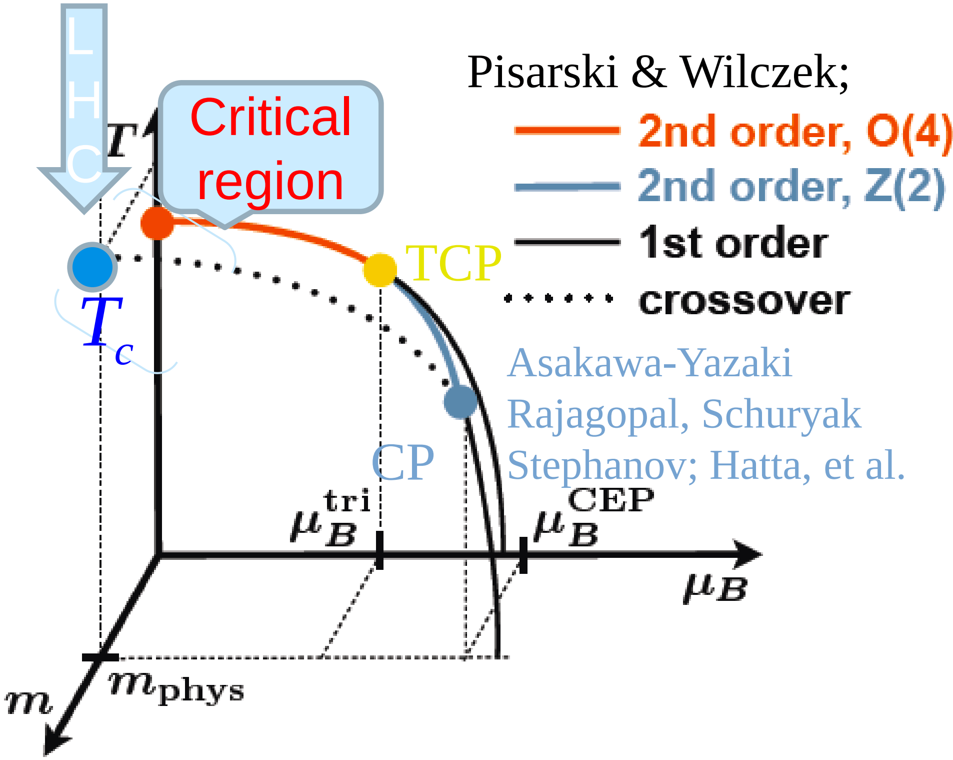


- QCD Phase Diagram from LQCD
- Fluctuations of conserved charges as probe of thermalization and QCD phase boundary

⇨ Linking LQCD results to HIC data at the LHC

⇨ STAR data on net-proton number fluctuations and chiral criticality

Deconfinement and chiral symmetry restoration in QCD



- The QCD chiral transition is crossover Y.Aoki, et al Nature (2006)
  and appears in the O(4) critical region

  O. Kaczmarek et.al. Phys.Rev. D83, 014504 (2011)

- Chiral transition temperature

  $$T_c = 155(1)(8) \text{ MeV}$$

  T. Bhattacharya et.al.
  Phys. Rev. Lett. 113, 082001 (2014)

- Deconfinement of quarks sets in at the chiral crossover

  A.Bazavov, Phys.Rev. D85 (2012) 054503

- The shift of T_c with chemical potential

  $$\boxed{T_c(\mu_B) = T_c(0)[1 - 0.0066 \cdot (\mu_B / T_c)^2]}$$

  Ch. Schmidt Phys.Rev. D83 (2011) 014504

  O. Kaczmarek QM 2017

See also:
Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, *et al.*
JHEP, 0906 (2009)

O(4) scaling and magnetic equation of state

$$P = P_R(T,\mu_q,\mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t, b^{\beta\delta/\nu} h)$$

- Phase transition encoded in the magnetic equation of state

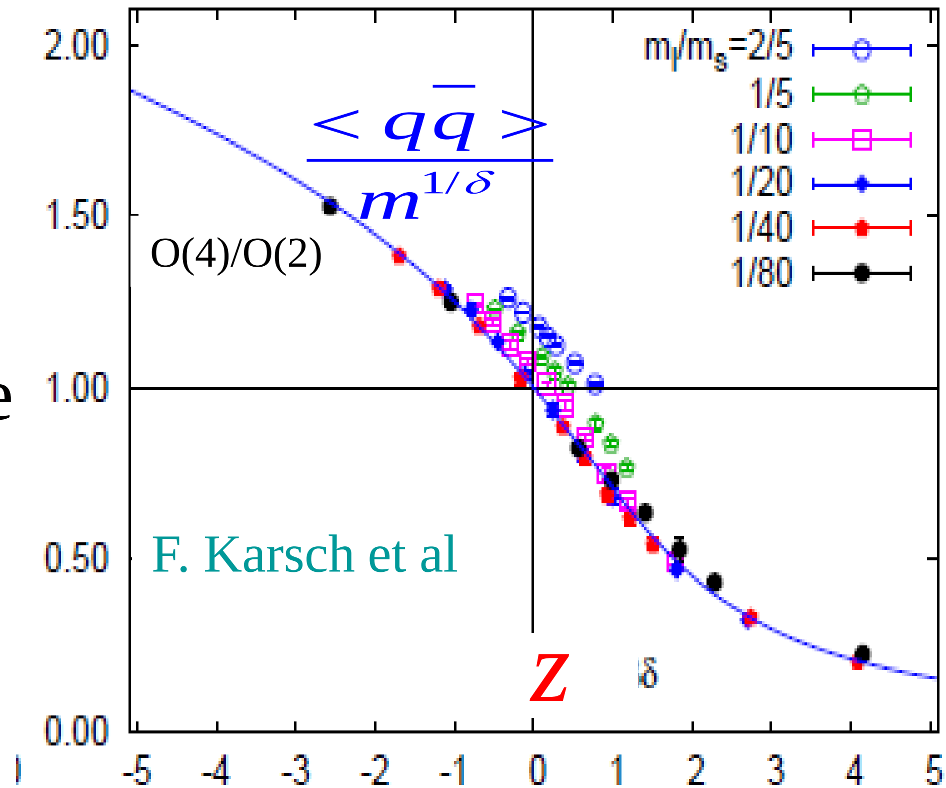
$$<q\bar{q}> = -\frac{\partial P}{\partial m} \Rightarrow \text{pseudo-critical line}$$

$$t = (T - T_c)/T_c$$

$$\boxed{\frac{<q\bar{q}>}{m^{1/\delta}} = f_s(z) \ , \quad z = t m^{-1/\beta\delta}}$$

universal scaling function common for all models belonging to the O(4) universality class: known from spin models J. Engels & F. Karsch (2012)

QCD chiral crossover transition in the critical region of the O(4) 2nd order



Probing O(4) chiral criticality with charge fluctuations

- Due to expected O(4) scaling in QCD the free energy:

$$\boxed{P = P_R(T,\mu_q,\mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t(\mu), b^{\beta\delta/\nu} h)}$$

- Generalized susceptibilities of net baryon number

$$\chi_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = \chi_R^{(n)} + \chi_S^{(n)} \quad \text{with} \quad \begin{cases} \chi_S^{(n)}|_{\mu=0} = d\ h^{(2-\alpha-n/2)/\beta\delta}\ f_{\pm}^{(n)}(z) \\ \chi_S^{(n)}|_{\mu\neq 0} = d\ h^{(2-\alpha-n)/\beta\delta}\ f_{\pm}^{(n)}(z) \end{cases}$$

- At $\mu = 0$ only $\chi_B^{(n)}$ with $n \geq 6$ receive contribution from $\chi_S^{(n)}$
- At $\mu \neq 0$ only $\chi_B^{(n)}$ with $n \geq 3$ receive contribution from $\chi_S^{(n)}$

$\chi_B^{n=2}$ generalized susceptibilities of the net baryon number is non critical with respect to O(4)

Consider fluctuations and correlations of conserved charges to be compared with LQCD

Excellent probe of:

- QCD criticality

A. Asakawa at. al.
S. Ejiri et al.,…
M. Stephanov et al.,
K. Rajagopal et al.
B. Frimann et al.

- freezeout conditions in HIC

F. Karsch &
S. Mukherjee et al.,
P. Braun-Munzinger et al.,,

- They are quantified by susceptibilities:

If $P(T,\mu_B,\mu_Q,\mu_S)$ denotes pressure, then

$$\frac{\chi_N}{T^2} = \frac{\partial^2(P)}{\partial(\mu_N)^2} \qquad \frac{\chi_{NM}}{T^2} = \frac{\partial^2(P)}{\partial\mu_N\partial\mu_M}$$

$$N = N_q - N_{-q}, \quad N,M = (B,S,Q), \quad \mu = \mu/T, \quad P = P/T^4$$

- Susceptibility is connected with variance

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(<N^2> - <N>^2)$$

- If $\mathrm{P}(N)$ probability distribution of N then

$$<N^n> = \sum_N N^n \mathrm{P}(N)$$

Consider special case:

$$< N_q > \equiv \overline{N}_q$$ =>

Charge carrying by particles $q = \pm 1$

- Charge and anti-charge uncorrelated and Poisson distributed, then
- $\mathrm{P}(N)$ the Skellam distribution

$$\mathrm{P}(N) = \left(\frac{\overline{N}_q}{\overline{\overline{N}}_{-q}} \right)^{N/2} I_N(2\sqrt{\overline{N}_{-q}\overline{N}_q}) \exp[-(\overline{N}_{-q} + \overline{N}_q)]$$

- Then the susceptibility

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N_q > + < N_{-q} >)$$

Consider special case: particles carrying $q = \pm 1, \pm 2, \pm 3$

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

- The probability distribution

$$< S_{-q} > \equiv \overline{S}_{-q}$$

$$q = \pm 1, \pm 2, \pm 3$$

$$\begin{aligned} P(S) \;=\; & \left(\frac{\bar{S}_1}{\bar{S}_{\bar{1}}}\right)^{\frac{S}{2}} \exp\left[\sum_{n=1}^{3} (\bar{S}_n + \bar{S}_{\bar{n}})\right] \\ & \sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left(\frac{\bar{S}_3}{\bar{S}_{\bar{3}}}\right)^{k/2} I_k\left(2\sqrt{\bar{S}_3 \bar{S}_{\bar{3}}}\right) \\ & \left(\frac{\bar{S}_2}{\bar{S}_{\bar{2}}}\right)^{i/2} I_i\left(2\sqrt{\bar{S}_2 \bar{S}_{\bar{2}}}\right) \\ & \left(\frac{\bar{S}_1}{\bar{S}_{\bar{1}}}\right)^{-i-3k/2} I_{2i+3k-S}\left(2\sqrt{\bar{S}_1 \bar{S}_{\bar{1}}}\right) \end{aligned}$$

Fluctuations

$$\frac{\chi_S}{T^2} = \frac{1}{VT^3} \sum_{n=1}^{|q|} n^2 \left(\langle S_n \rangle + \langle S_{-n} \rangle\right)$$

Correlations

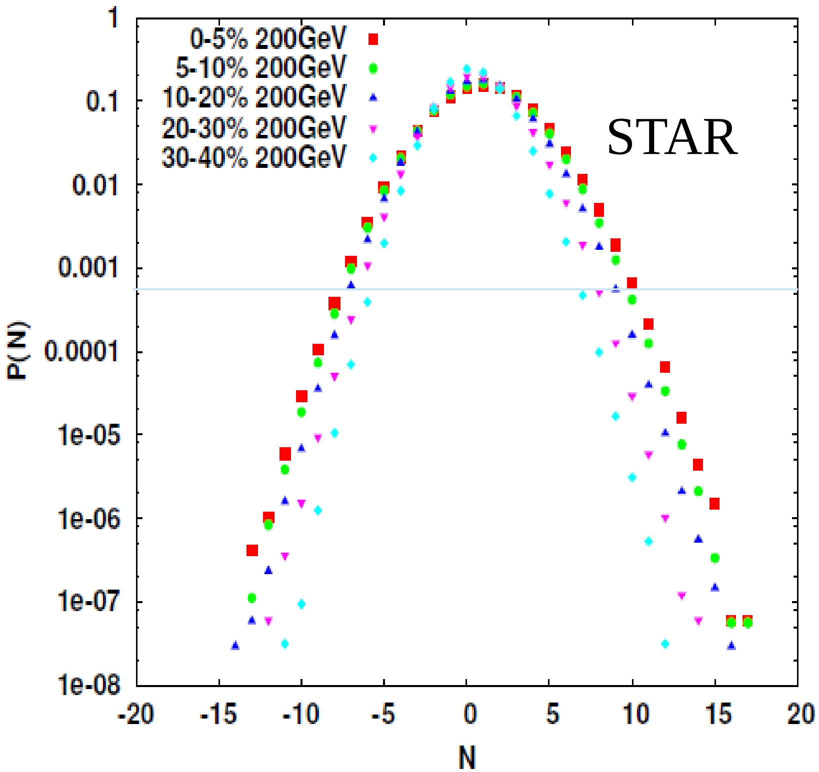
$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3} \sum_{m=-q_M}^{q_M} \sum_{n=-q_N}^{q_N} nm \langle S_{n,m} \rangle$$

$\langle S_{n,m} \rangle$:is the mean number of particles carrying charge $N = n$ and $M = m$

Variance at 200 GeV AA central coll. at RHIC

STAR Collaboration data in central coll. 200 GeV

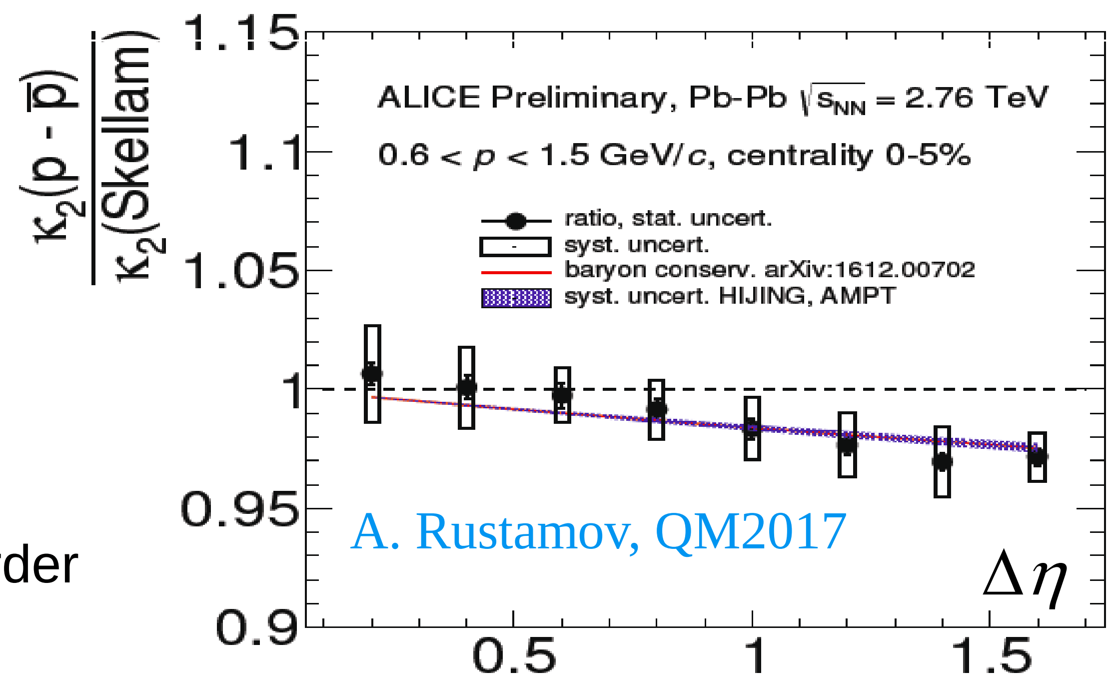
See also talk of Lijun Ruan



- Consistent with Skellam distribution

$$\boxed{\frac{<p>+<\overline{p}>}{\sigma^2}=1.022\pm0.016} \qquad \frac{\chi_1}{\chi_3}=1.076\pm0.035$$

- ALICE data consistent with Skellam $\Delta\eta<1$



- Skellam distribution is a good approximation to calculate 2nd order charge fluctuations in HIC

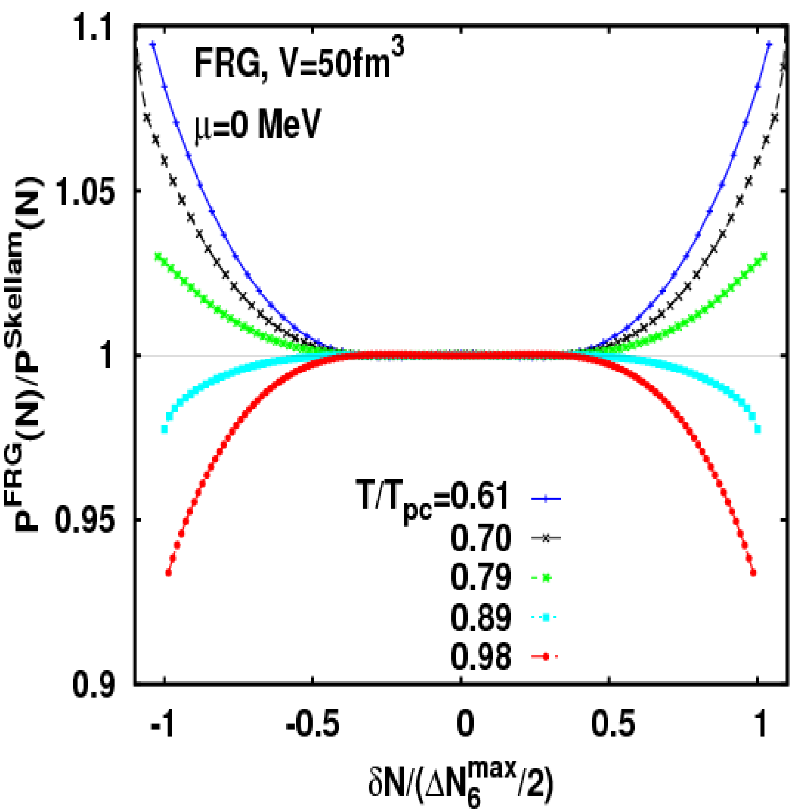
Variance at 200 GeV AA central coll. at RHIC

K. Morita, B. Friman and K.R. Phys.Lett. B741 (2015) 178

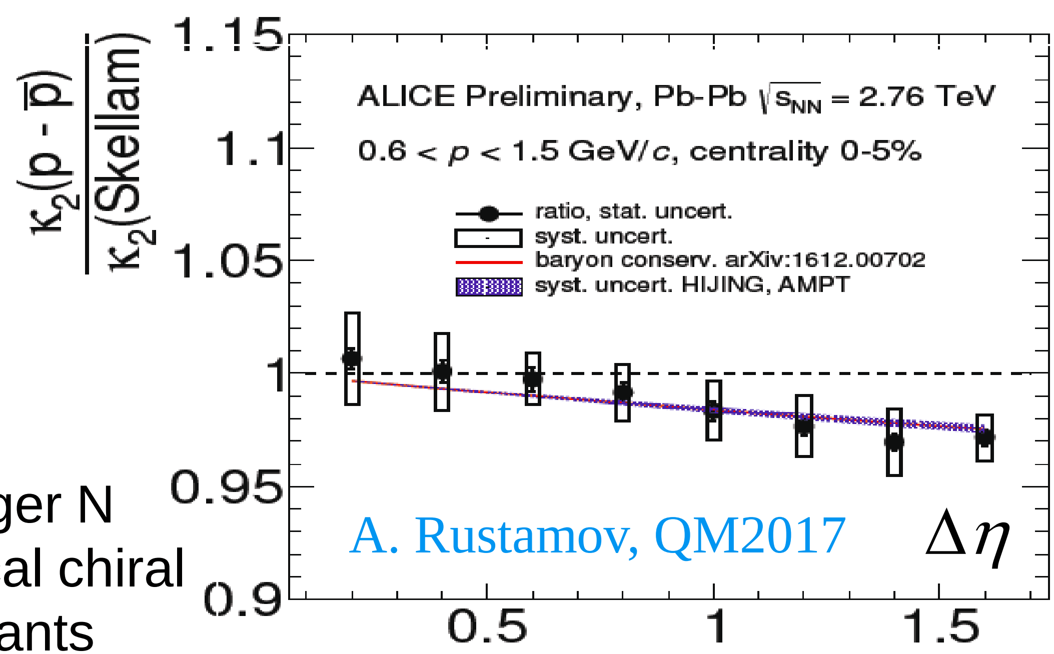
STAR Collaboration data in central coll. 200 GeV

Consistent with Skellam distribution

$$\frac{<p>+<\bar{p}>}{\sigma^2} = 1.022 \pm 0.016 \qquad \frac{\chi_1}{\chi_3} = 1.076 \pm 0.035$$



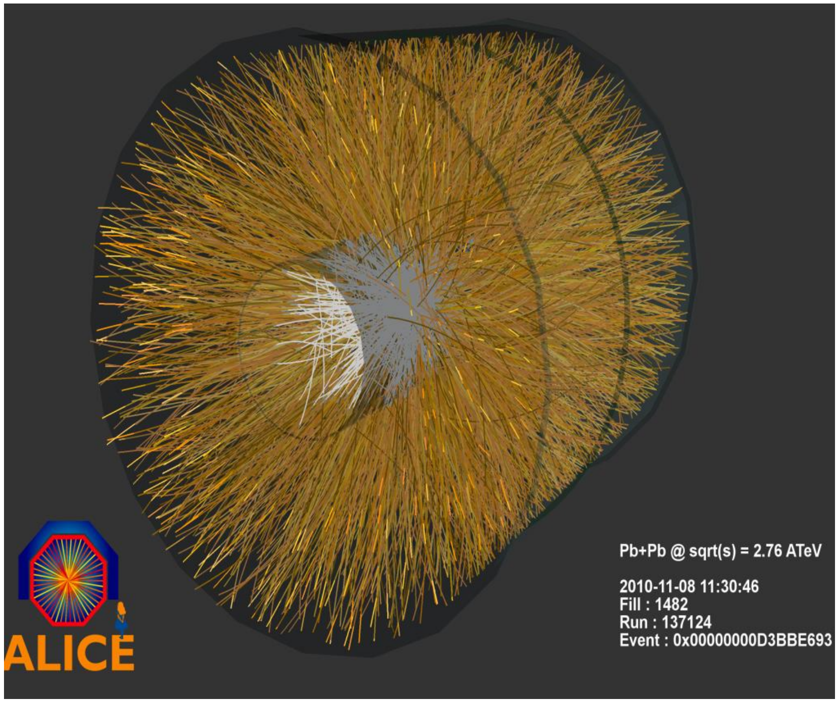
ALICE data consistent with Skellam $\Delta\eta < 1$

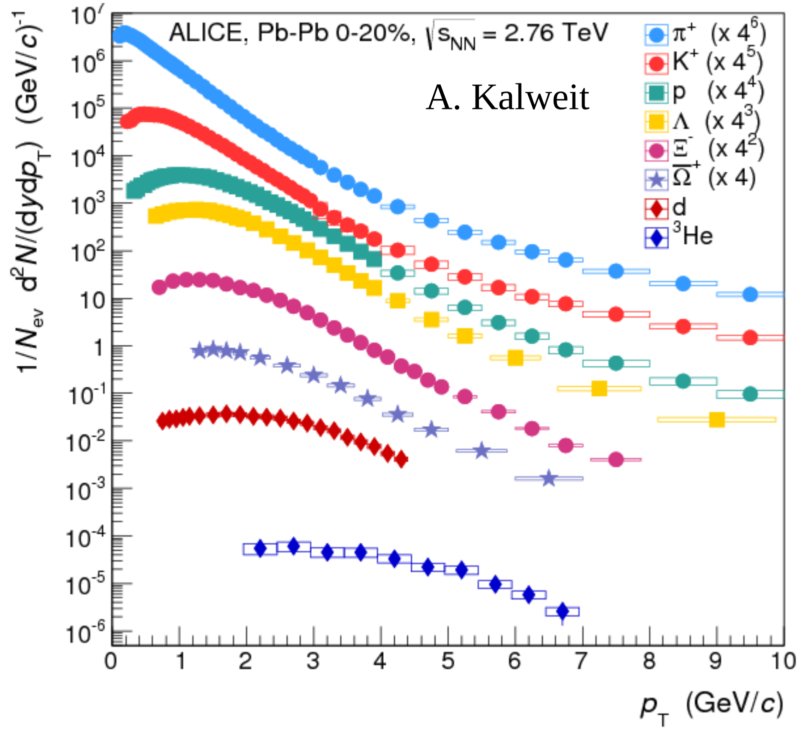


- Shrinking of Skellam dis. at larger N needed to capture the O(4) critical chiral properties of higher order cumulants

Thermal particle prodution in Heavy Ion Collisions form SIS to LHC

ALICE Collaboration





Can the thermal nature and composition of the collision fireball in HIC be verified ?

Constructing net charge fluctuations and correlation from ALICE data with Skellam distribution

- Net baryon number susceptibility

$$\frac{\chi_B}{T^2} \approx \frac{1}{VT^3}(\langle p\rangle + \langle N\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + \langle \Xi^-\rangle + \langle \Xi^0\rangle + \langle \Omega^-\rangle + \overline{par}\ )$$

- Net strangeness

$$\frac{\chi_S}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + \langle K_S^0\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + 4\langle \Xi^-\rangle + 4\langle \Xi^0\rangle + 9\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-} + \Gamma_{\varphi\to K_S^0} + \Gamma_{\varphi\to K_L^0})\langle \varphi\rangle\ )$$

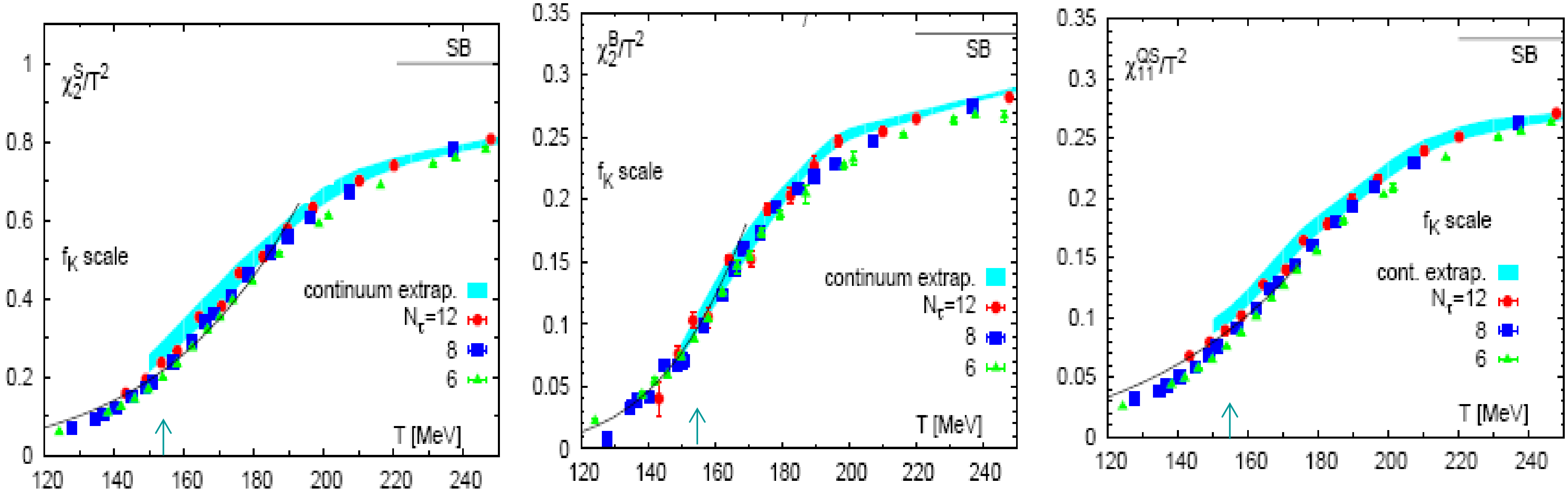
- Charge-strangeness correlation

$$\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + 2\langle \Xi^-\rangle + 3\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-})\langle \varphi\rangle - (\Gamma_{K_0^*\to K^+} + \Gamma_{K_0^*\to K^-})\langle K_0^*\rangle\ )$$

Compare the ratio with LQCD data:

A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa and S. Mukherjee

Phys.Rev.Lett. 113 (2014) and HotQCD Coll. A. Bazavov et al. Phys.Rev. D86 (2012) 034509



- Is there a temperature where calculated ratios from ALICE data agree with LQCD?

Direct comparisons of Heavy ion data at LHC with LQCD

- STAR results => the 2nd order cumulants χ_2 are consistent with Skellam distribution, thus χ_N and χ_{NM} with $N,M=\{B,Q,S\}$ are expressed by particle yields. Consider LHC data

$$\frac{\chi_B}{T^2}=\frac{1}{VT^3}(203.7\pm 11.4)$$

$$\frac{\chi_S}{T^2}=\frac{1}{VT^3}(504.2\pm 16.8)$$

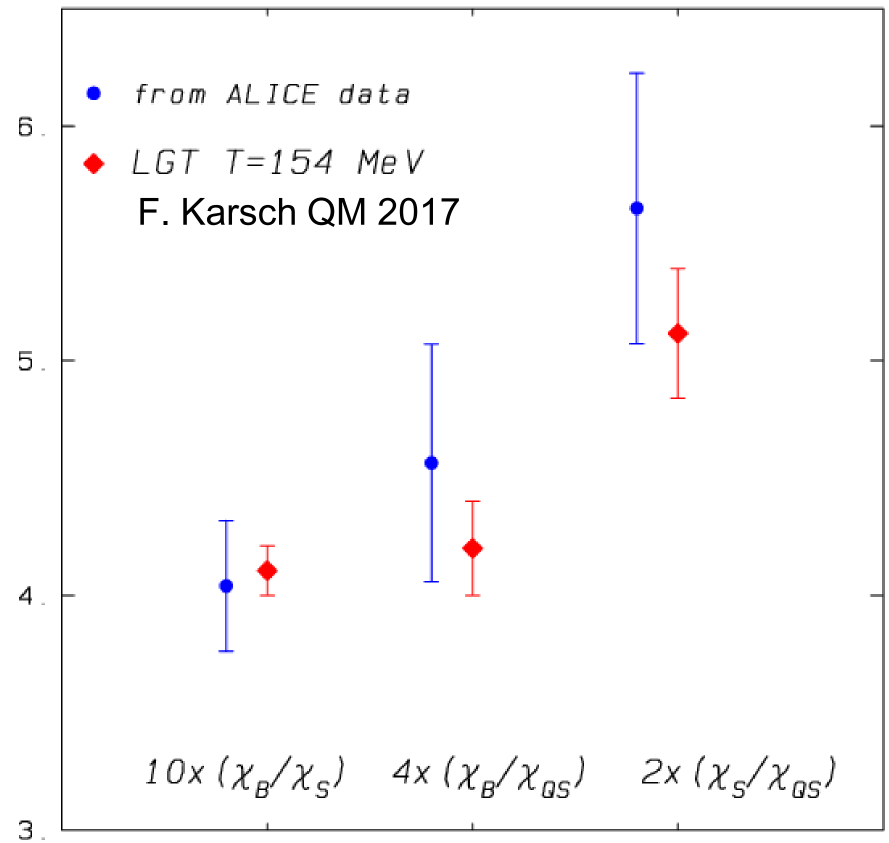
$$\frac{\chi_{QS}}{T^2}=\frac{1}{VT^3}(191.1\pm 12)$$

- The Volume at T_c

$$V_{T_c}=3800\pm 500\ fm^3$$

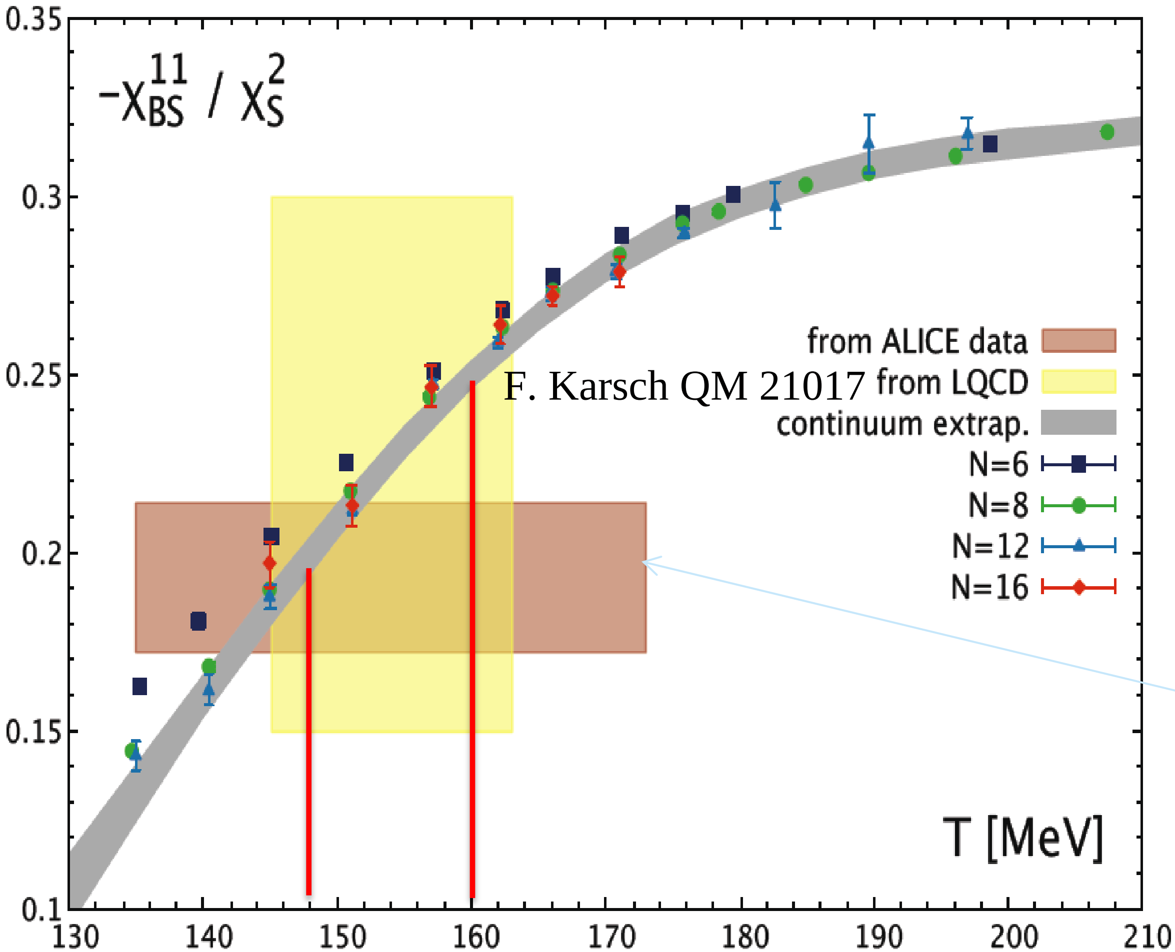
Compare ratios with LQCD at chiral crossover

P. Braun-Munzinger, A. Kalweit, J. Stachel, & K.R. Phys. Lett. B747, 292 (2015)



The cumulant ratios extracted from ALICE data are consistent with LQCD at the chiral crossover: Evidence for thermalization at the phase boundary

Constraining chemical freezeout temperature at the LHC



At the LHC energy the fireball created in HIC is a QCD medium at the chiral cross over temperature.

$$C_{BS} = -\frac{<(\delta B)(\delta S)>}{<(\delta S)^2>} = -\frac{\chi_{BS}}{\chi_S}$$

- Excellent observable to fix the temperature

$$-\frac{\chi_{BS}}{T^2} \approx \frac{1}{VT^3}[2<\Lambda+\Sigma^0>+4<\Sigma^+>$$
$$+8<\Xi>+6<\Omega^->] = (97.4 \pm 5.8)/VT^3$$

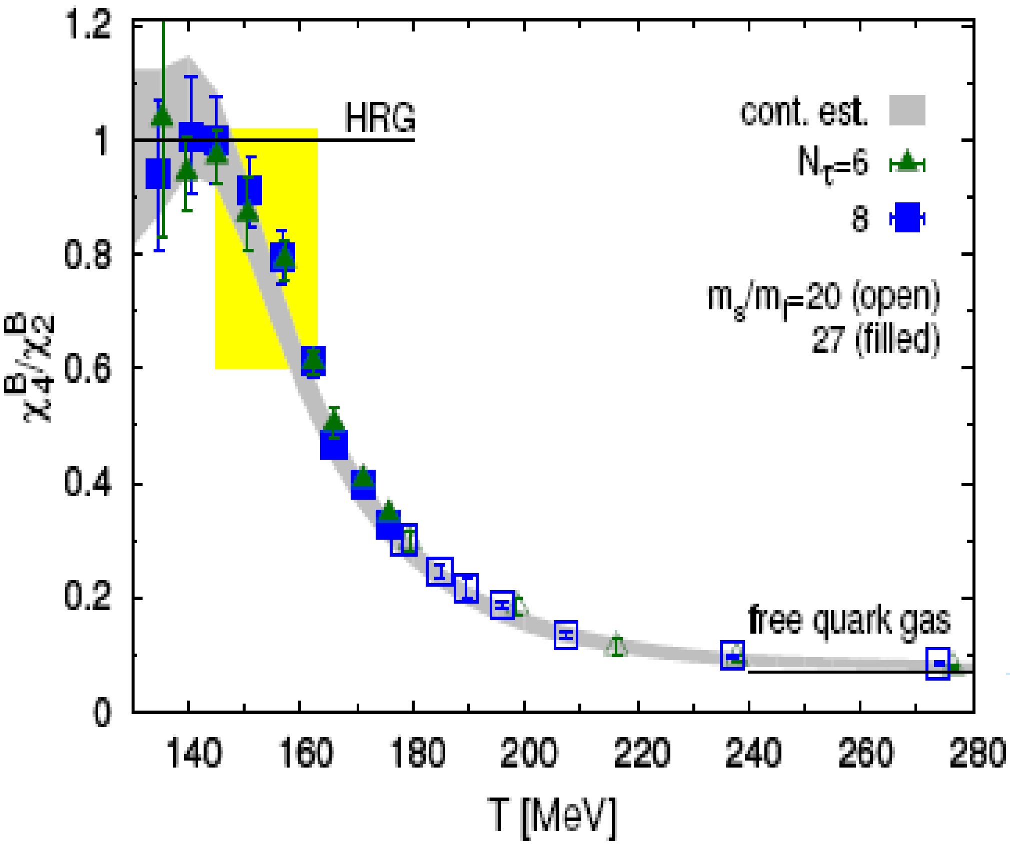
However, this is the lower limit since e.g. $\Sigma^*(\geq 1660) \to N\bar{K}$ $\Lambda^*(\geq 1520) \to N\bar{K}$ are not included

- Data on χ_B/χ_S and χ_B/χ_{QS} consistent with LQCD results for $0.148 \leq T_f < 160\ MeV$

Fluctuations of net baryon number sensitive to deconfinement in QCD

S. Ejiri, F. Karsch & K.R. (06)
A. Bazavov et al. arXiv. 1701.04325

$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$$



- HRG factorization of pressure:

$$\boxed{P^B(T,\mu_q) = F(T)\cosh(B\mu_B/T)}$$

- Kurtosis measures the squared of the baryon number carried by leading particles in a medium

S. Ejiri, F. Karsch & K.R. (06)

$\frac{1}{9}$

$$\kappa\,\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

Modelling fluctuations in the O(4)/Z(2) universality class

$$\mathscr{L}_{\rm QM} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - U(\sigma,\vec{\pi})$$

Effective potential is obtained by solving *the exact flow equation* (Wetterich eq.) with the approximations resulting in the O(4)/Z(2) <u>critical exponents</u>

B.J. Schaefer & J. Wambach,; B. Stokic, B. Friman & K.R.

$$\partial_k\Omega_k(\sigma) = \frac{Vk^4}{12\pi^2}\left[\sum_{i=\pi,\sigma}\frac{d_i}{E_{i,k}}[1+2n_B(E_{i,k})] - \frac{2\nu_q}{E_{q,k}}[1-n_F(E^+_{q,k})-n_F(E^-_{q,k})]\right]$$

$$E_{\pi,k} = \sqrt{k^2+\Omega'_k}$$

$$E_{\sigma,k} = \sqrt{k^2+\Omega'_k+2\rho\Omega''_k}$$

$$E_{q,k} = \sqrt{k^2+2g^2\rho}$$

$$\Omega'_k \equiv \frac{\partial\Omega_k}{\partial(\sigma^2/2)}$$

Full propagators with k < q < Λ

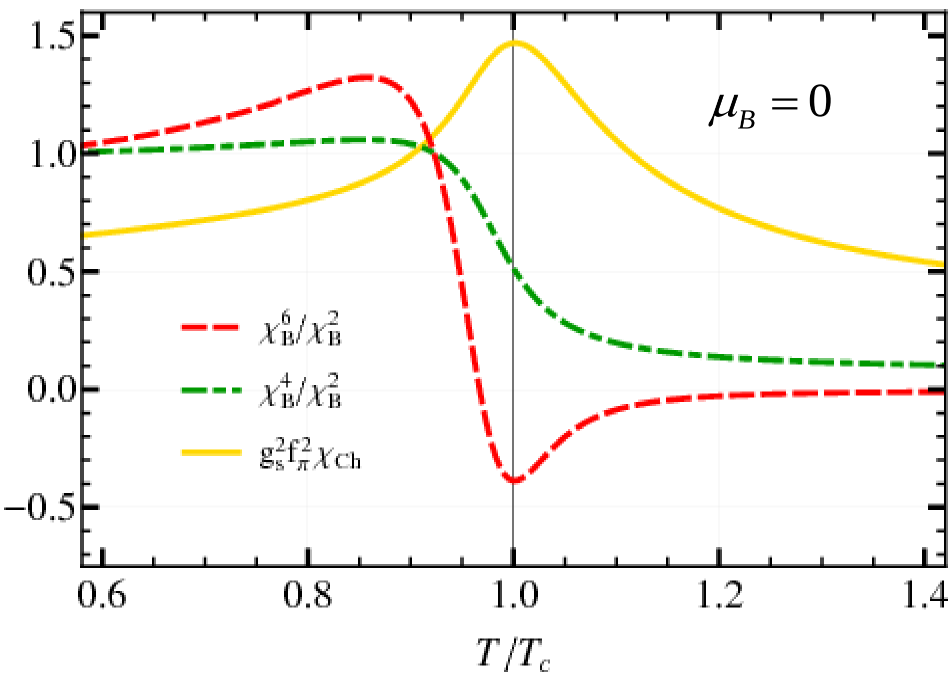
$\Gamma_\Lambda = S_{\text{classical}}$

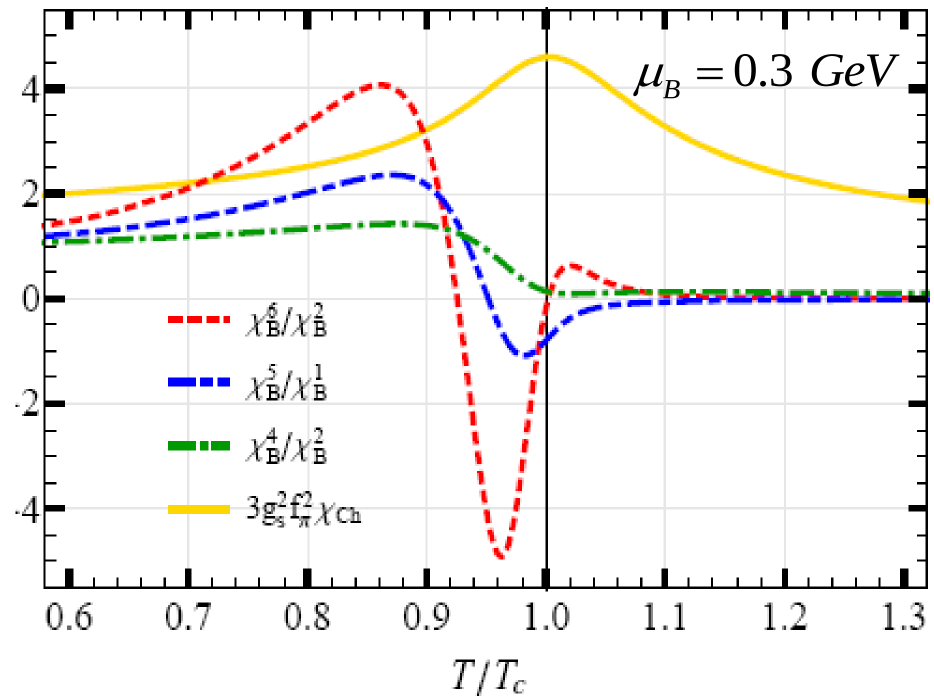
Integrating from k=Λ to k=0 gives full quantum effective potential

Higher order cumulants in effective chiral model within FRG approach, belongs to O(4)/Z(2) universality class

B. Friman, V. Skokov &K.R. Phys. Rev. C83 (2011) 054904
G. Almasi, B. Friman &K.R. arXiv:1703.05947

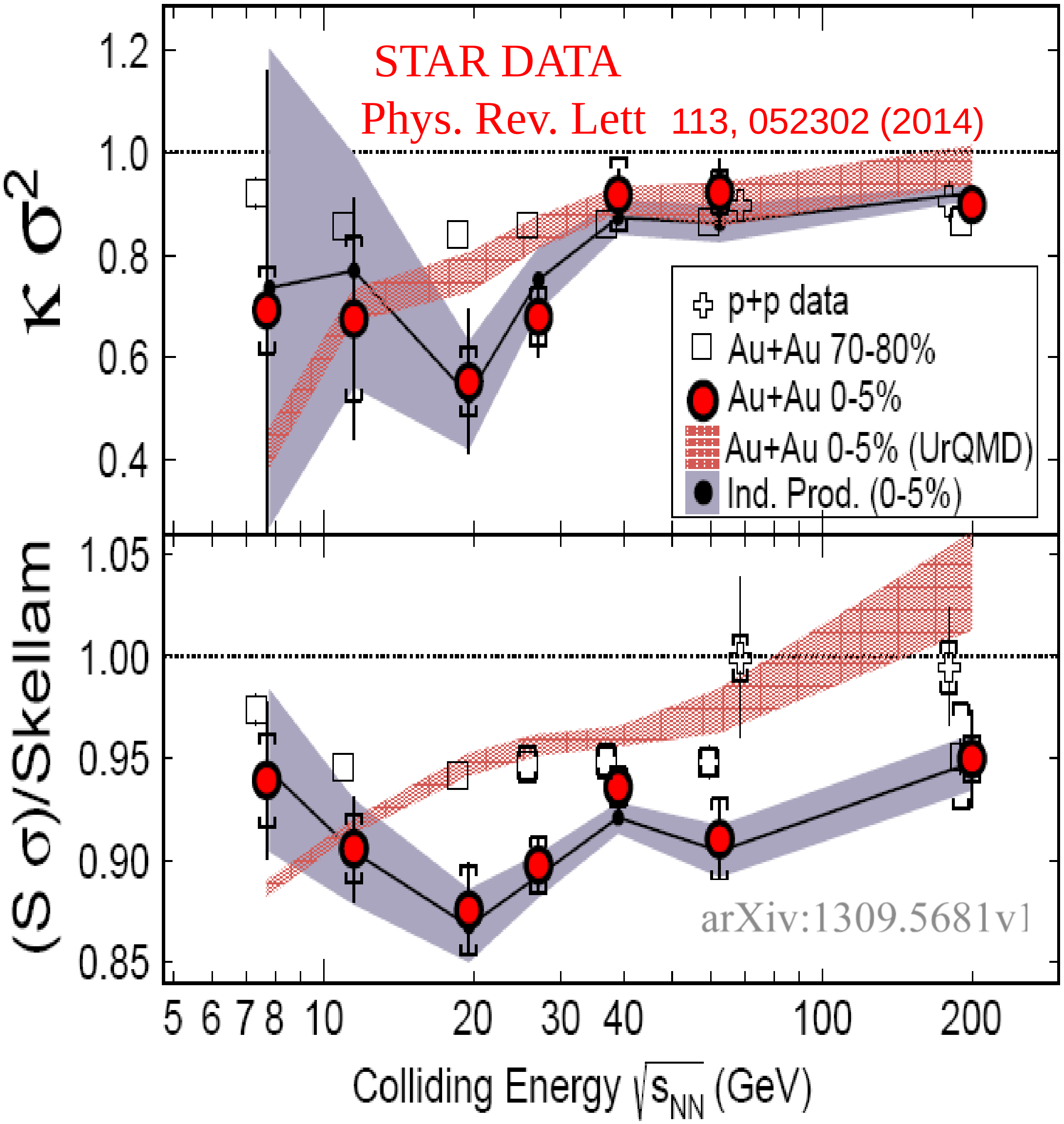
G. Almasi, B. Friman &K.R. arXiv:1703.05947





Deviations of cumulant ratios from Skellam distribution are increasing with the order of the cumulants and can be used to identify the chiral QCD phase boundary in HIC

STAR data on the cumulants of the net baryon number



Deviations from the HRG

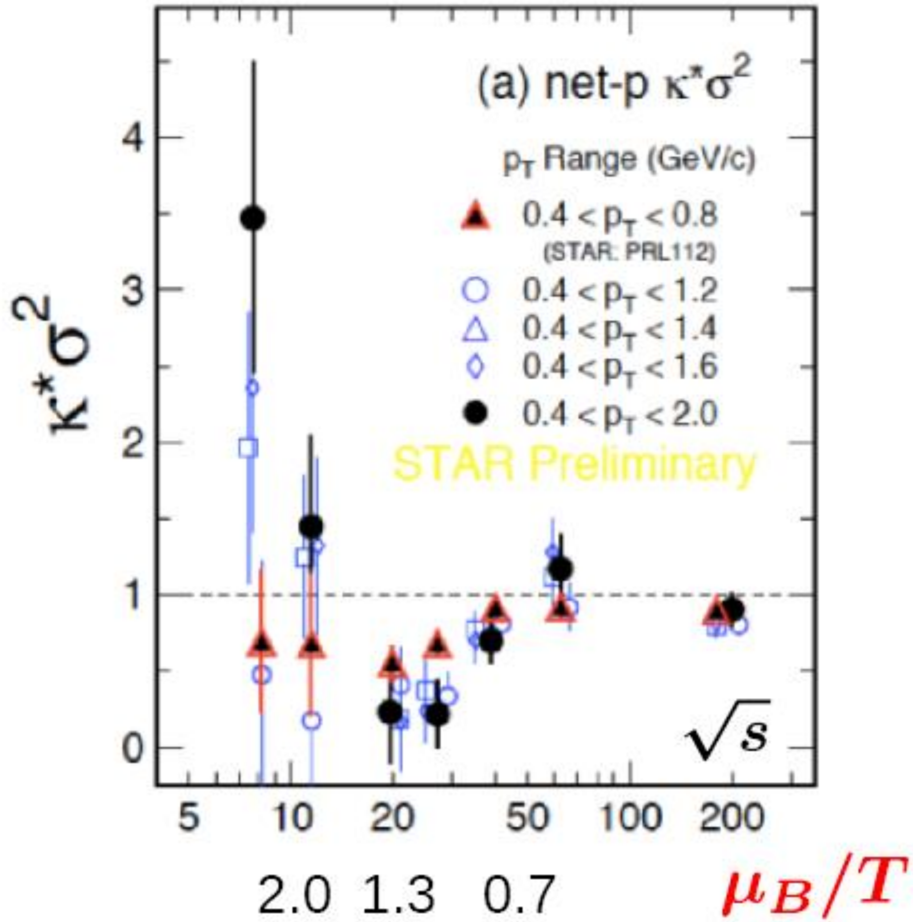
$$S\sigma = \frac{\chi_B^{(3)}}{\chi_B^{(2)}} \;, \quad \kappa\sigma^2 = \frac{\chi_B^{(4)}}{\chi_B^{(2)}}$$

$$S\sigma|_{HRG} = \frac{N_p - N_{\bar{p}}}{N_p + N_{\bar{p}}} \;, \kappa\sigma|_{HRG} = 1$$

Data qualitatively consistent with the change of these ratios due to the contribution of the O(4) singular part to the free energy

STAR “BES” and recent results on net-proton fluctuations

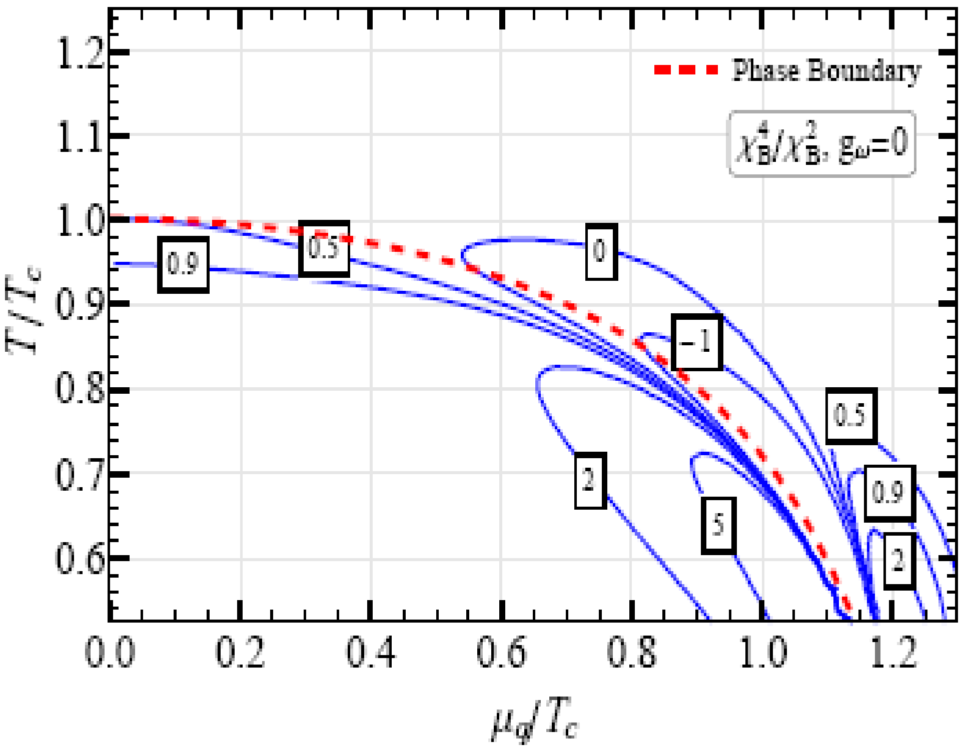
X. Luo et al. (2015), STAR Coll. Preliminary



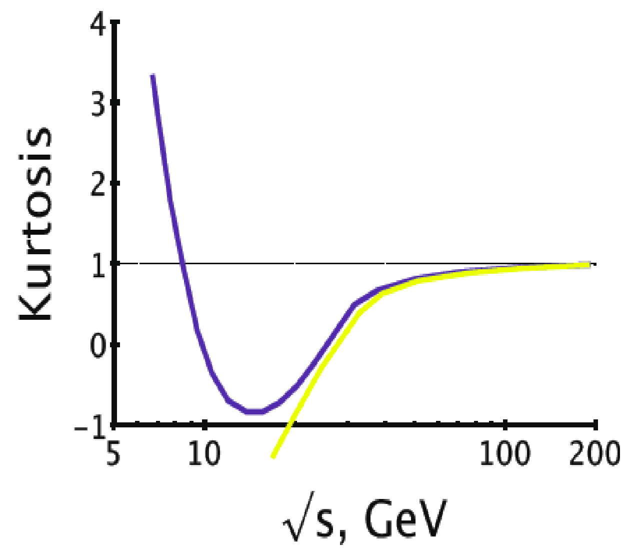
See also talk of Lijun Ruan

- With increasing acceptance of the transverse momentum, large increase of net-proton fluctuations at $\sqrt{s} < 20$ GeV beyond that of a non-critical reference of a HRG
- Is the above an Indication of the CEP?
- At $\sqrt{s} > 20$ GeV data consistent with LQCD results near the chiral crossover

Modelling critical fluctuations



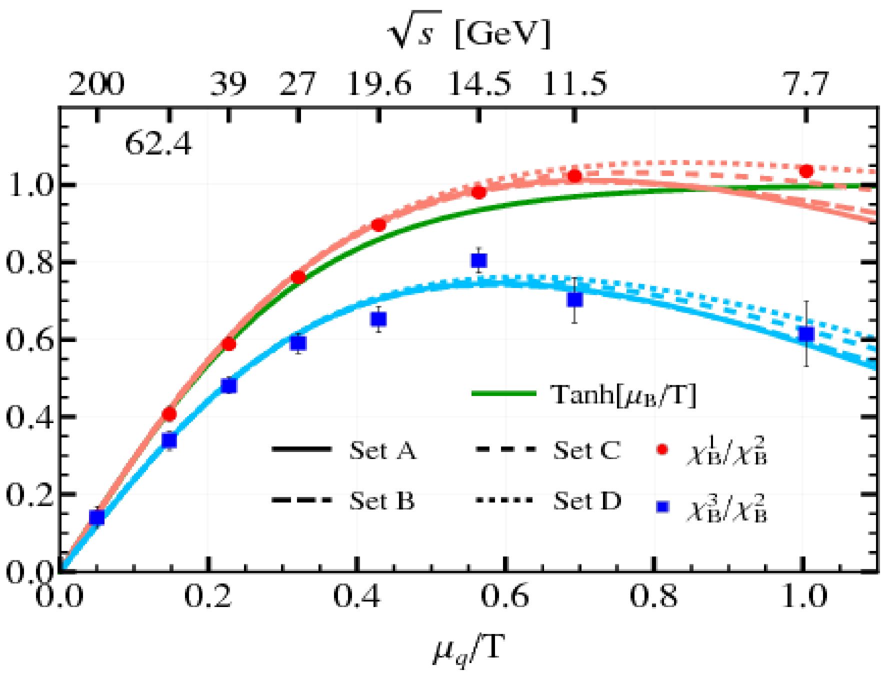
- It is possible to find the freeze-out line such that kurtosis exhibits the energy dependence as seen in data.



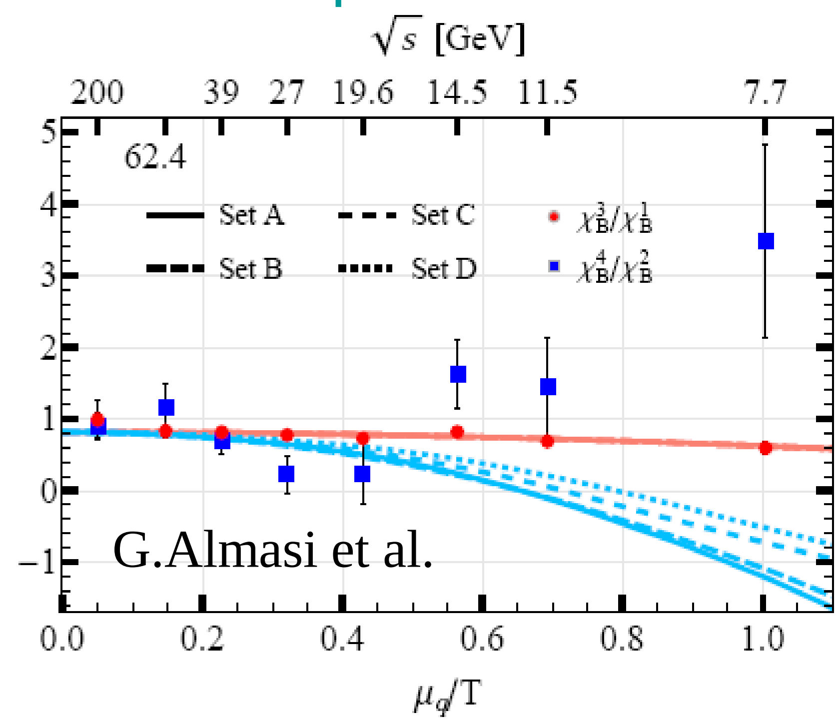
- However, are other cumulants consistent?

Self - consistent freeze-out and STAR data

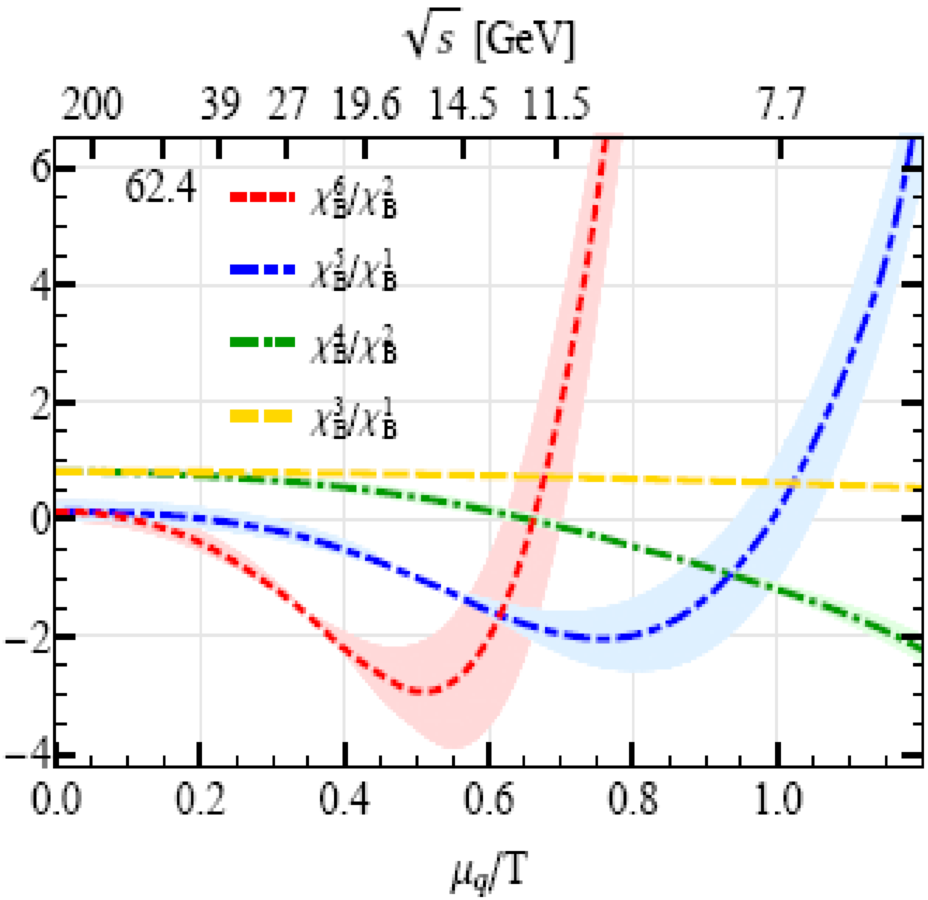
- Freeze-out line in $(T,\mu)-$ plain determined by fitting χ_B^3/χ_B^1 to data
- Ratio $\chi_B^1/\chi_B^2 \approx \tanh(\mu/T)$ => further evidence of equilibrium and thermalisation at $7\ GeV \le \sqrt{s} < 5\ TeV$
- Ratio $\chi_B^1/\chi_B^2 \neq \chi_B^3/\chi_B^2$ expected due to critical chiral dynamics
- Enhancement of χ_B^4/χ_B^2 at $\sqrt{s} < 20\ GeV$ not reproduced



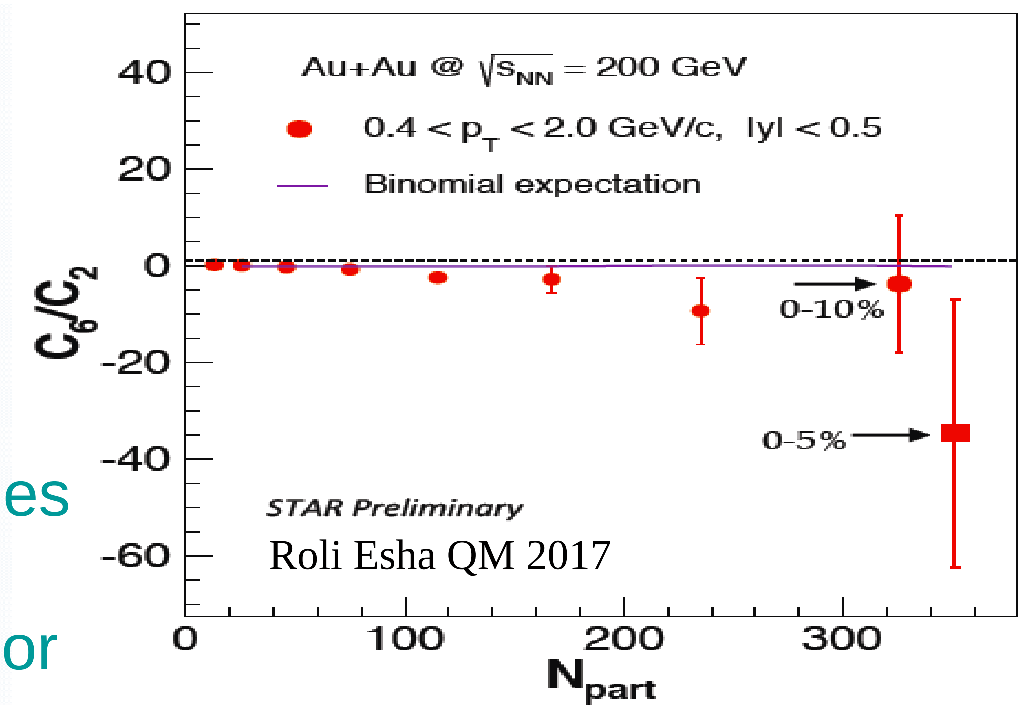
Gabor Almasi, Bengt Friman & K.R.



Higher order cumulants - energy dependence



- Strong non-monotonic variation of higher order cumulants at lower $\sqrt{s}$
- Equality of different ratios excellent probes of equilibrium evolution in HIC



- At freeze-out fixed by χ_B^3/χ_B^1, the ratio $\chi_B^2/\chi_B^2 \approx 0$ which agrees with preliminary STAR data, albeit within still very large error

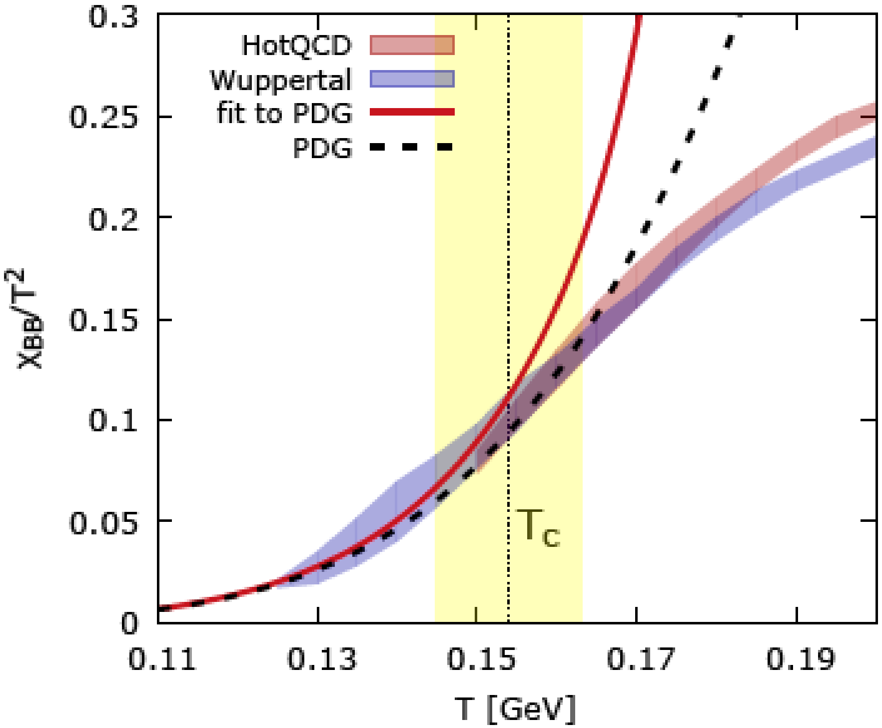
Conclusions:

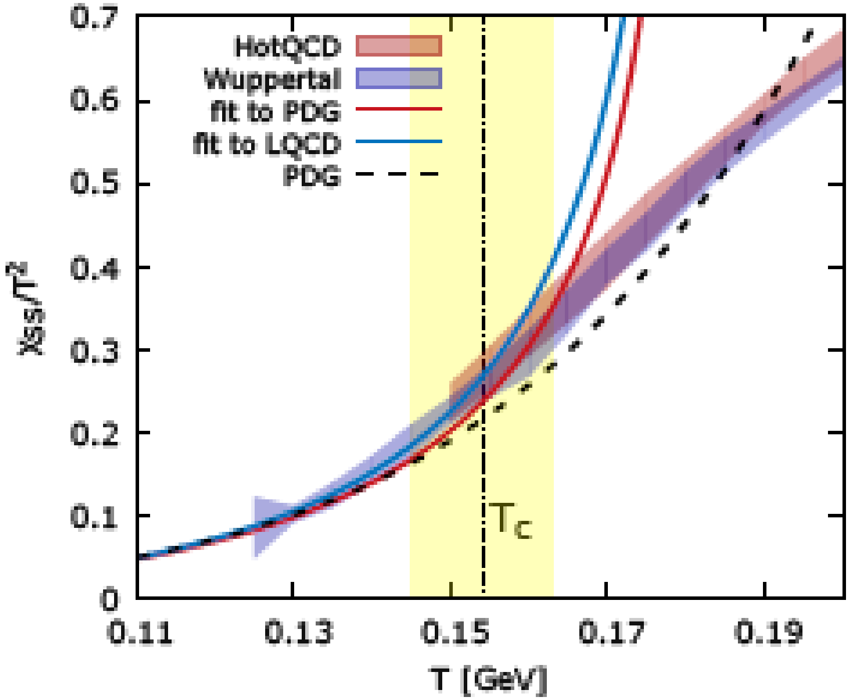
- From LQCD: chiral crossover in QCD is the remnant of the 2^{nd} order phase transition belonging to the O(4) universality class

Very good prospects for exploring the phase diagram of QCD in nuclear collisions with fluctuations

- The medium created in HIC is of thermal origin and follows the properties expected in LQCD near the phase boundary
- Systematics of the net-proton number fluctuations at $\sqrt{s} \geq 20$ GeV measured by STAR Coll. in HIC at RHIC is qualitatively consistent with the expectation, that they are influenced by the critical chiral dynamics

Hagedorn's continuum mass spectrum contribution to strangeness fluctuations





- Missing strange baryon and meson resonances in the PDG

F. Karsch, et al., Phys. Rev. Lett. 113, no. 7, 072001 (2014)

P.M. Lo, et al. Eur.Phys.J. A52 (2016)

- Satisfactory description of LGT with asymptotic states from Hagedorn's exponential mass spectrum $\rho^H(m) = m^a e^{m/T_H}$ fitted to PDG

Charge - Strangeness correlations

- The ratio

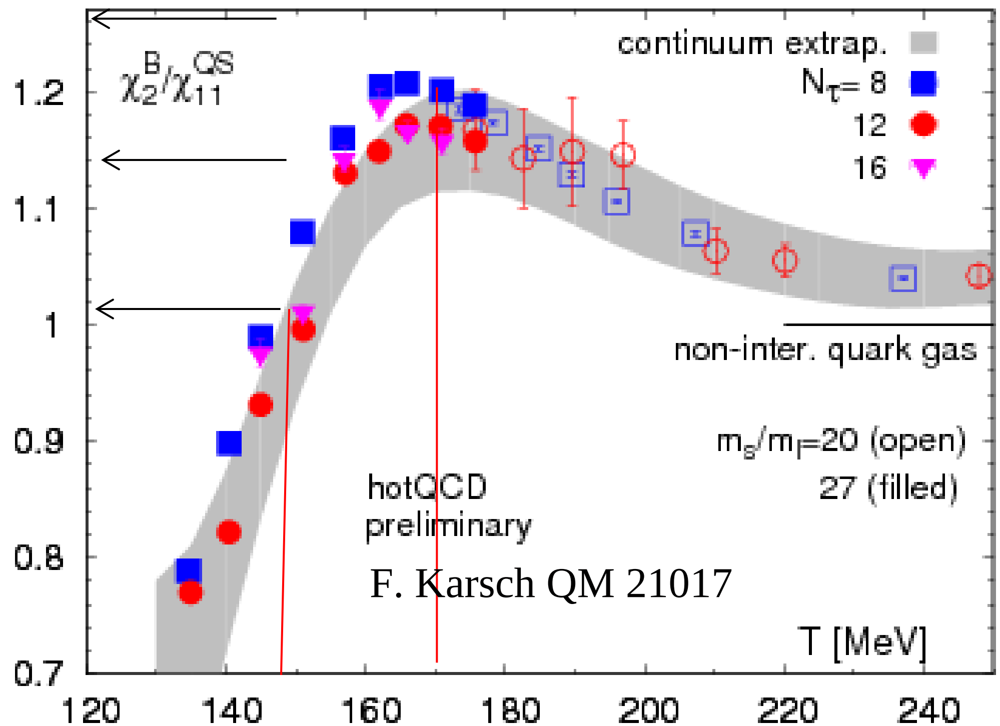
$$1.014 \leq \frac{\chi_2^B}{\chi_2^{QS}} \leq 1.267$$

extracted from ALICE data is consistent with LQCD for

$$148 < T_f \leq 170 \quad \text{MeV}$$

when combined with T_f obtained from χ_2^B / χ_2^S one concludes that, data consistent with LGT for

$$148 < T_f \leq 160$$



The ratio of cumulants in LGT and ALICE data

- The ratio

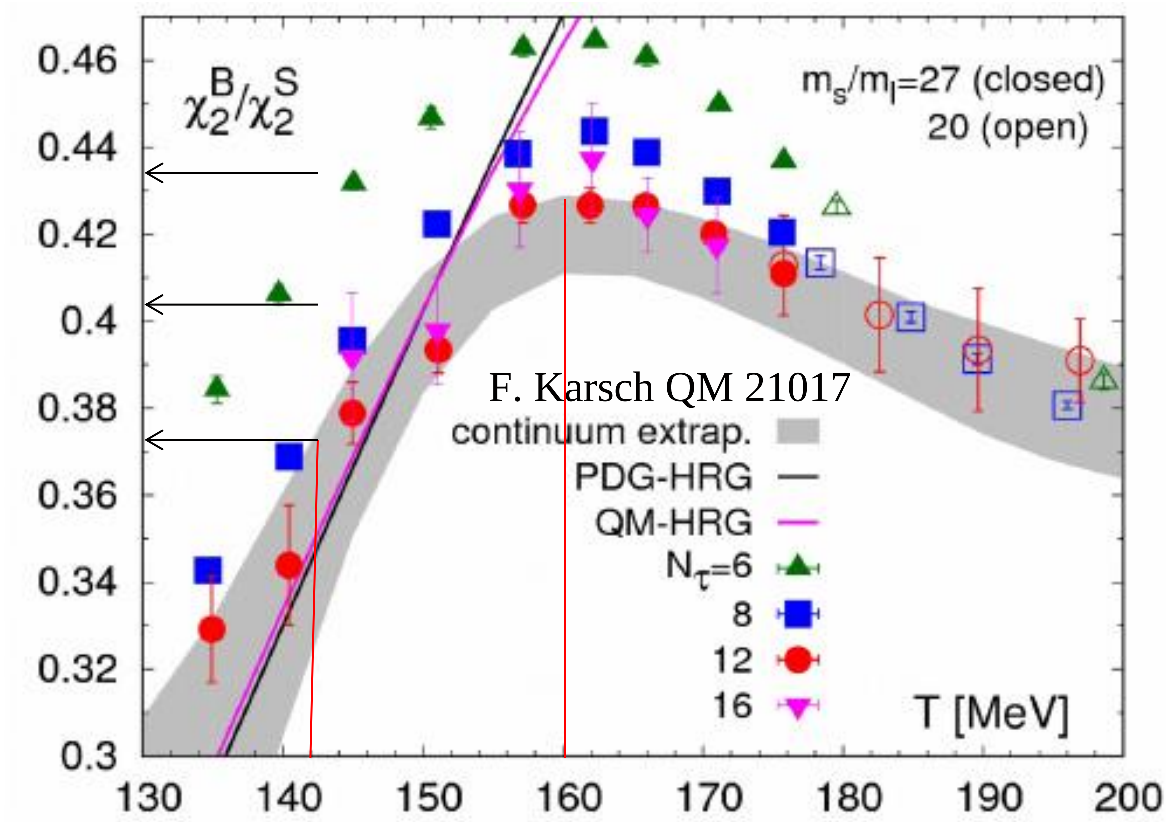
$$0.376 \le \frac{\chi_2^B}{\chi_2^S} \le 0.432$$

extracted from ALICE data is consistent with LQCD for

$$142 < T_f \le 160 \text{ MeV}$$

thus excellently overlaps with chiral crossover

$$145 < T_c \le 163 \text{ MeV}$$

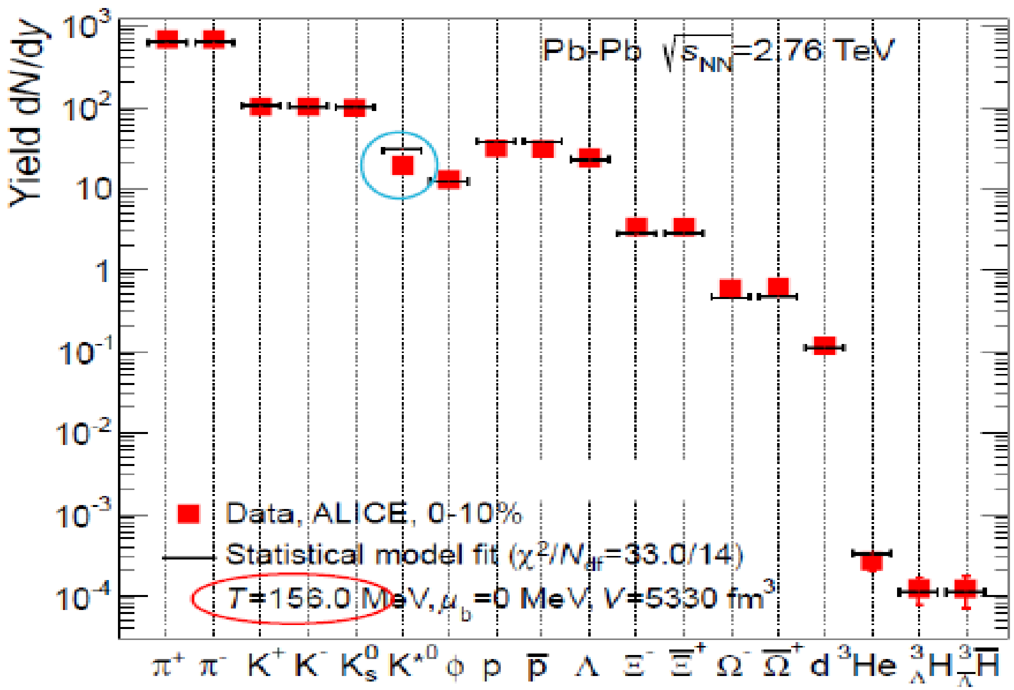


Modelling QCD Statistical Operator in hadronic phase

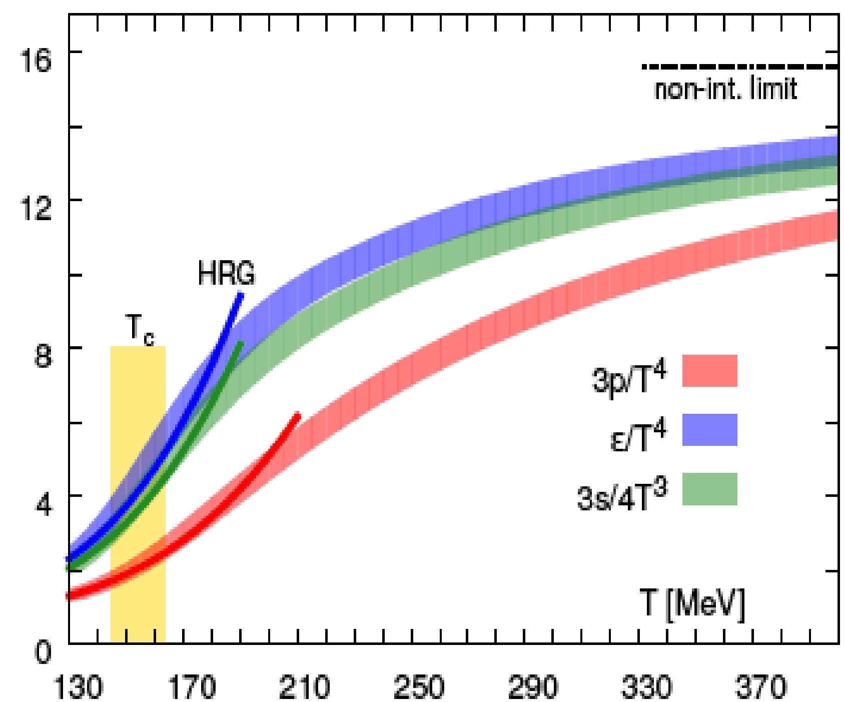
Interacting hadron gas => S matrix approach (Dashen, Ma & Bernstein, Phys. Rev (1969):
"uncorrelated" gas of hadrons and resonances (HRG)

$$P_H^{int.} \approx \sum_{\substack{i=stabel \\ hadrons}} P_i^{id} + \sum_{\substack{k=all \\ resonaces}} P_k^{id}$$

A. Andronic, Peter Braun-Munzinger, & Johanna Stachel, et al.



A. Bazavov et al. HotQCD Coll. 2014, 2017



HRG provides very good description of yields data and LQCD equation of state