One and two-nucleon distributions and the contact term – theory

Ronen Weiss and Nir Barnea

The Hebrew University of Jerusalem

Collaborators: O. Hen, E. Piasetzki and R. Torres

The atomic contact

- Zero-range condition: $r_0 \ll a, d$

Interaction range

Scattering length

Distance between particles

S. Tan, Ann. Phys. (N.Y.) 323, 2952 (2008); Ann. Phys. (N.Y.) 323, 2971 (2008); Ann. Phys. (N.Y.) 323, 2987 (2008)

The atomic contact

- Zero-range condition: $r_0 \ll a, d$

Interaction range

Scattering length

Distance between particles

- Many quantities are connected to the ***contact C***:

$$n(k) = \boldsymbol{C}/k^4 \text{ for } k \to \infty$$

$$T + U = \frac{\hbar^2}{4\pi m a}\boldsymbol{C} + \sum_\sigma \frac{d^3k}{(2\pi)^3}\frac{\hbar^2 k^2}{2m}\left(n_\sigma(\boldsymbol{k}) - \frac{\boldsymbol{C}}{k^4}\right)$$

and many more…

S. Tan, Ann. Phys. (N.Y.) 323, 2952 (2008); Ann. Phys. (N.Y.) 323, 2971 (2008); Ann. Phys. (N.Y.) 323, 2987 (2008)

The atomic contact

- The basic **factorization** assumption:

$$\psi \xrightarrow{r_{ij}\to 0} \left(\frac{1}{r_{ij}} - \frac{1}{a}\right) \times A\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

The atomic contact

- The basic **factorization** assumption:

$$\psi \xrightarrow{r_{ij}\to 0} \left(\frac{1}{r_{ij}} - \frac{1}{a}\right) \times A\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

$$n(k) \to \frac{1}{k^4} \times \boldsymbol{C}$$

The atomic contact

- The basic **factorization** assumption:

$$\psi \xrightarrow{r_{ij}\to 0} \left(\frac{1}{r_{ij}} - \frac{1}{a}\right) \times A\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

$$n(k) \to \frac{1}{k^4} \times \boldsymbol{C}$$

NOT FOR NUCLEAR PHYSICS

$$r_0 \ll d, a$$

The Nuclear contacts

$$\psi \xrightarrow{r_{ij}\to 0} \left(\frac{1}{r_{ij}} - \frac{1}{a}\right) \times A\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

$$\Downarrow$$

$$\psi \xrightarrow{r_{ij}\to 0} \varphi_{ij}(r_{ij}) \times A\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

The Nuclear contacts

$$\psi \xrightarrow{r_{ij}\to 0} \left(\frac{1}{r_{ij}} - \frac{1}{a}\right) \times A\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

$$\Downarrow$$

$$\psi \xrightarrow{r_{ij}\to 0} \varphi_{ij}(r_{ij}) \times A\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

$$\Downarrow$$

$$\psi \xrightarrow{r_{ij\to 0}} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) \times A_{ij}^{\alpha}\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

Channels α
$= (\ell_2 S_2) j_2 m_2$

"universal"
function

The pair kind
$ij \in \{pp, nn, pn\}$

The Nuclear contact – Momentum

One-body momentum distribution – $\boldsymbol{n_N(k)}$ – The probability to find a proton/neutron with momentum k

Two-body momentum distribution – $\boldsymbol{F_{NN}(k)}$ – The probability to find an NN pairs with relative momentum k

The Nuclear contact – Momentum

One-body momentum distribution – $\boldsymbol{n_N(k)}$ – The probability to find a proton/neutron with momentum k

Two-body momentum distribution – $\boldsymbol{F_{NN}(k)}$ – The probability to find an NN pairs with relative momentum k

$$\psi \xrightarrow{r_{ij\to 0}} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) \times A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

$$n_p(\boldsymbol{k}) \xrightarrow{\mathrm{k}\to\infty} \sum_{\alpha,\beta} \left[\tilde{\varphi}_{pp}^{\alpha\,\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pp}^{\beta}(\boldsymbol{k}) 2 C_{pp}^{\alpha\beta} + \tilde{\varphi}_{pn}^{\alpha\,\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pn}^{\beta}(\boldsymbol{k}) C_{pn}^{\alpha\beta} \right]$$

The Nuclear contact – Momentum

One-body momentum distribution – $\boldsymbol{n_N(k)}$ – The probability to find a proton/neutron with momentum k

Two-body momentum distribution – $\boldsymbol{F_{NN}(k)}$ – The probability to find an NN pairs with relative momentum k

$$\psi \xrightarrow{r_{ij\to 0}} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) \times A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

$$n_p(\boldsymbol{k}) \xrightarrow{\mathrm{k}\to\infty} \sum_{\alpha,\beta} \left[\tilde{\varphi}_{pp}^{\alpha\,\dagger}(\boldsymbol{k})\tilde{\varphi}_{pp}^{\beta}(\boldsymbol{k}) 2C_{pp}^{\alpha\beta} + \tilde{\varphi}_{pn}^{\alpha\,\dagger}(\boldsymbol{k})\tilde{\varphi}_{pn}^{\beta}(\boldsymbol{k}) C_{pn}^{\alpha\beta}\right]$$

$$F_{ij}(\boldsymbol{k}) \xrightarrow{\mathrm{k}\to\infty} \sum_{\alpha,\beta} \tilde{\varphi}_{ij}^{\alpha\,\dagger}(\boldsymbol{k})\tilde{\varphi}_{ij}^{\beta}(\boldsymbol{k})\, C_{ij}^{\alpha\beta}$$

The Nuclear contact – Momentum

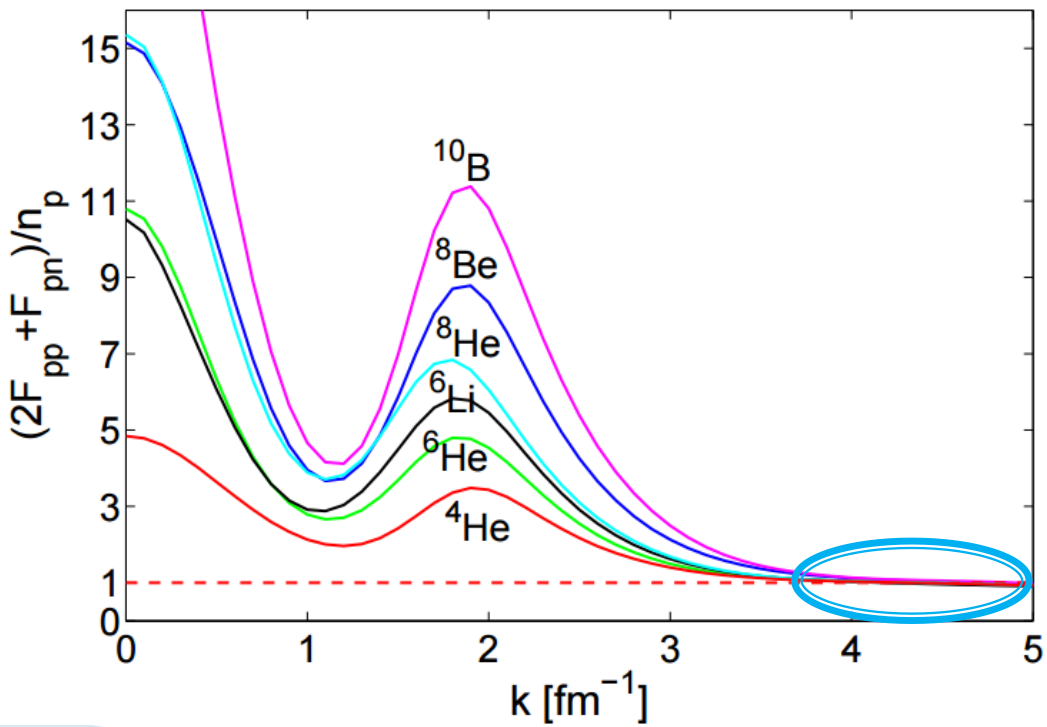
- As a result we get the asymptotic relation:

$$n_p(\boldsymbol{k}) \to F_{pn}(\boldsymbol{k}) + 2F_{pp}(\boldsymbol{k})$$

The Nuclear contact – Momentum

- As a result we get the asymptotic relation:

$$n_p(\boldsymbol{k}) \rightarrow F_{pn}(\boldsymbol{k}) + 2F_{pp}(\boldsymbol{k})$$



Using the variational Monte Carlo data (VMC)

Wiringa et al. Phys. Rev. C 89, 024305 (2014)

Extracting the contacts

- Assuming only **two significant channels**:

  The **deuteron** channel – L=0,2; S=1; J=1; T=0

  The **pure s-wave** channel – L=0; S=0; J=0; T=1

- We get:

$$F_{pn}(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^{2} + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^{2}$$

$$F_{nn}(k) \xrightarrow[k\to\infty]{} C_{nn}^{0}\left|\varphi_{nn}^{0}(k)\right|^{2}$$

Extracting the contacts

- Assuming only **two significant channels**:

  The **deuteron** channel – L=0,2; S=1; J=1; T=0

  The **pure s-wave** channel – L=0; S=0; J=0; T=1

- We get:

$$F_{pn}(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^{2} + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^{2}$$

$$F_{nn}(k) \xrightarrow[k\to\infty]{} C_{nn}^{0}\left|\varphi_{nn}^{0}(k)\right|^{2}$$

The VMC data

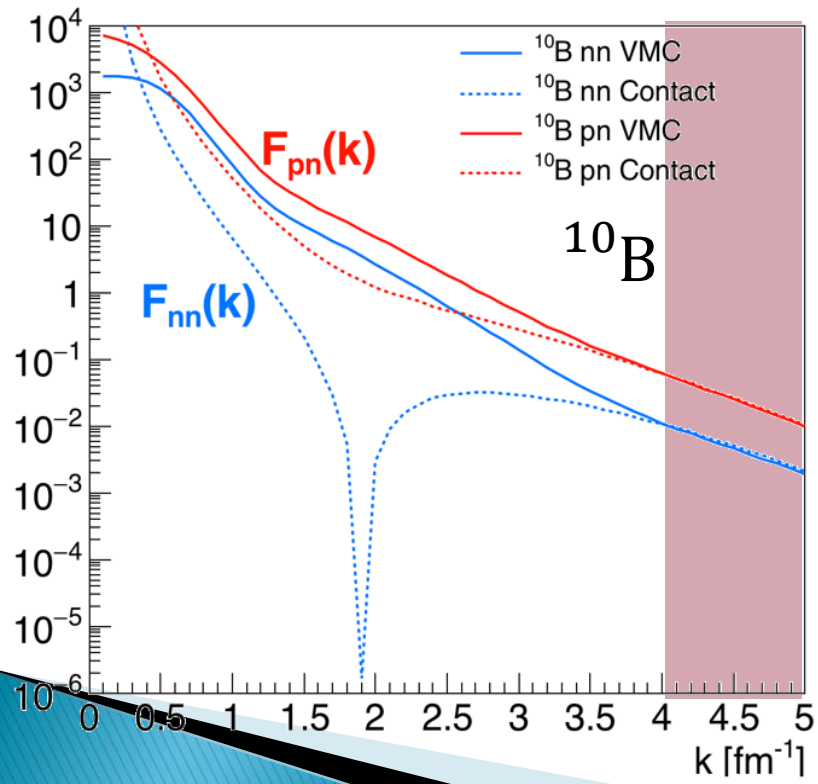
Zero-energy solution of the two-body system (AV18)

Extracting the contacts

$$F_{pn}(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2$$

$$F_{nn}(k) \xrightarrow[k\to\infty]{} C_{nn}^{0}\left|\varphi_{nn}^{0}(k)\right|^2$$

Momentum space

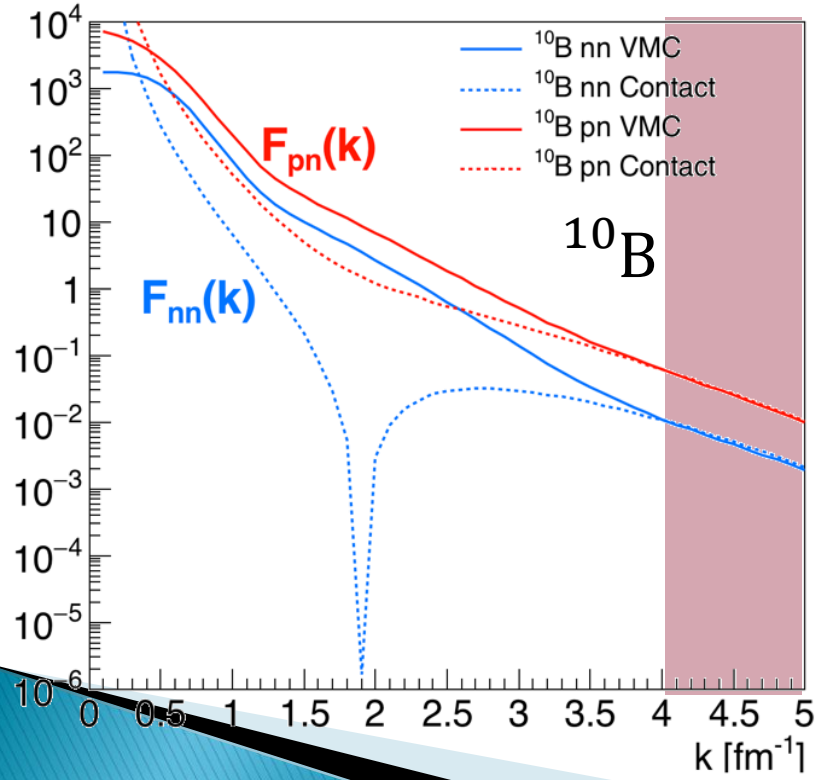


Extracting the contacts

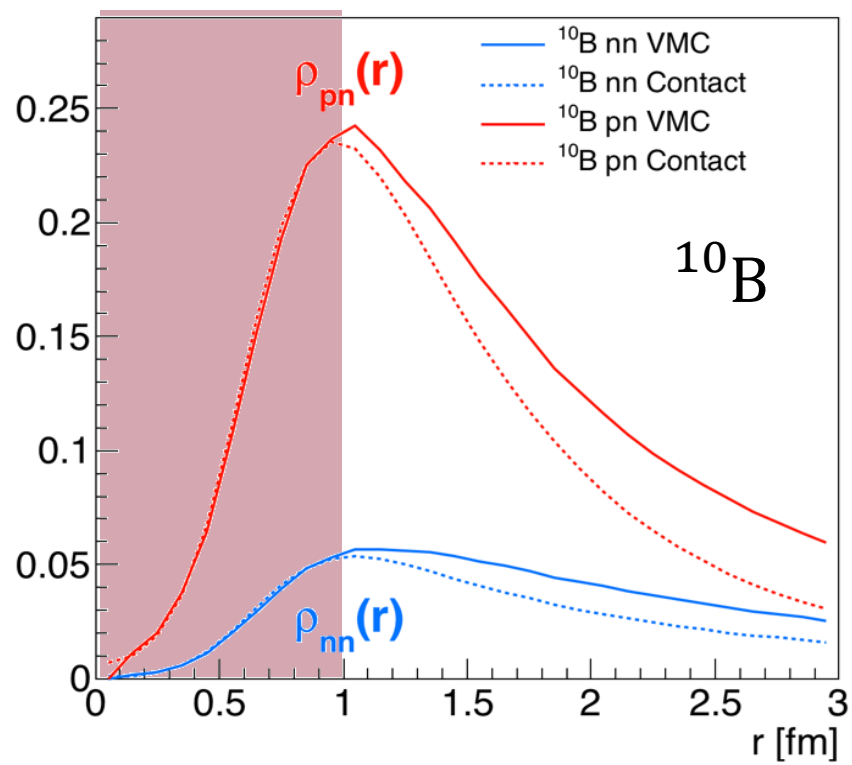
$$F_{pn}(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2$$

$$F_{nn}(k) \xrightarrow[k\to\infty]{} C_{nn}^{0}\left|\varphi_{nn}^{0}(k)\right|^2$$

Momentum space



Coordinate space



Extracting the contacts

$$n_p(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2 + 2C_{pp}^{0}\left|\varphi_{pp}^{0}(k)\right|^2$$

Universal functions – Calculated for the two-body system

Extracting the contacts

$$n_p(k) \xrightarrow[k\to\infty]{} C^d_{pn}\left|\varphi^d_{pn}(k)\right|^2 + C^0_{pn}\left|\varphi^0_{pn}(k)\right|^2 + 2C^0_{pp}\left|\varphi^0_{pp}(k)\right|^2$$

Fitted to $F_{ij}(k)$ for $k > 4\ \mathrm{fm}^{-1}$

Extracting the contacts

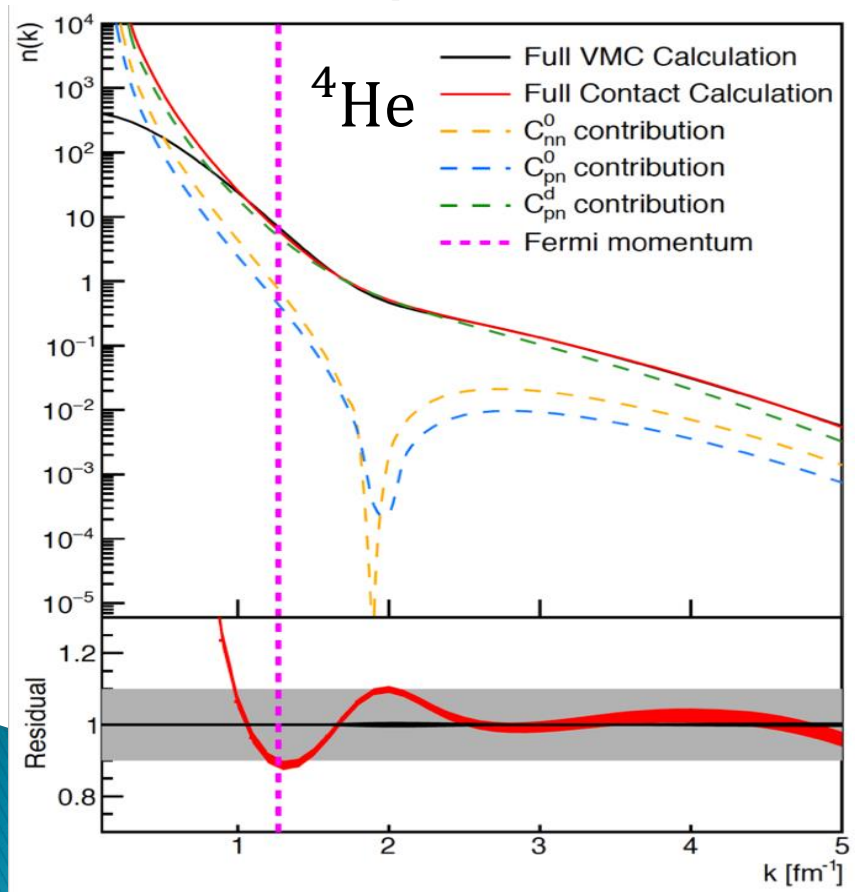
$$n_p(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2 + 2C_{pp}^{0}\left|\varphi_{pp}^{0}(k)\right|^2$$

The VMC data

Extracting the contacts

$$n_p(k) \xrightarrow[k\to\infty]{} C^d_{pn}\left|\varphi^d_{pn}(k)\right|^2 + C^0_{pn}\left|\varphi^0_{pn}(k)\right|^2 + 2C^0_{pp}\left|\varphi^0_{pp}(k)\right|^2$$

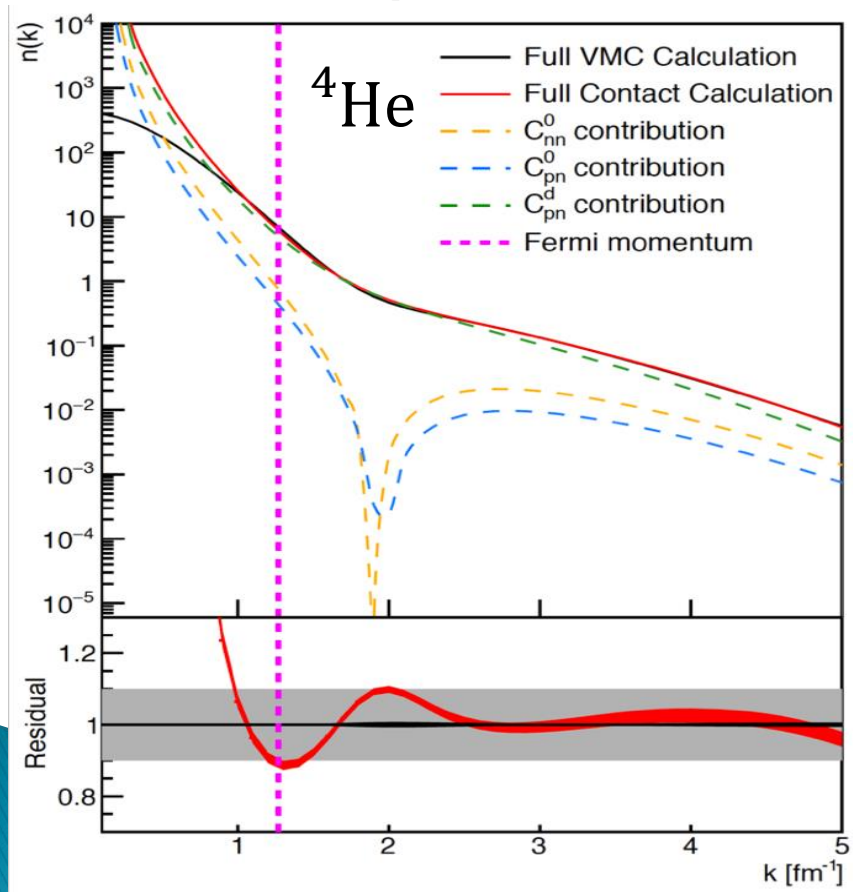
$n_p(k)$



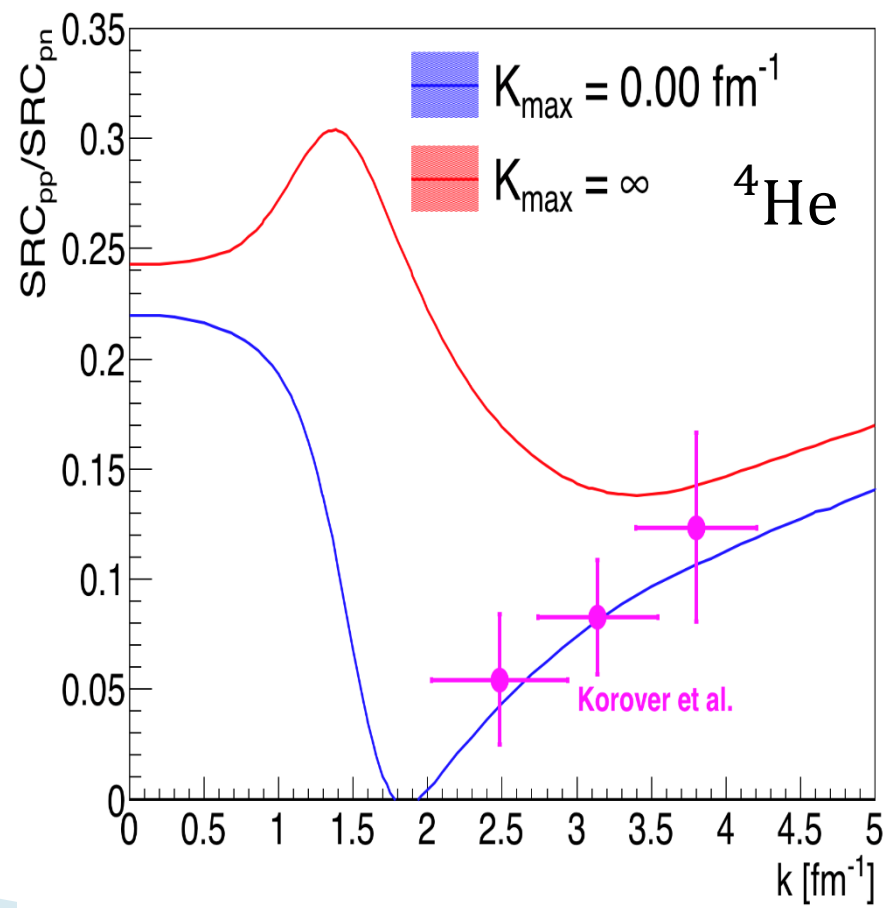
Extracting the contacts

$$n_p(k) \xrightarrow[k\to\infty]{} C^d_{pn}\left|\varphi^d_{pn}(k)\right|^2 + C^0_{pn}\left|\varphi^0_{pn}(k)\right|^2 + 2C^0_{pp}\left|\varphi^0_{pp}(k)\right|^2$$

$n_p(k)$



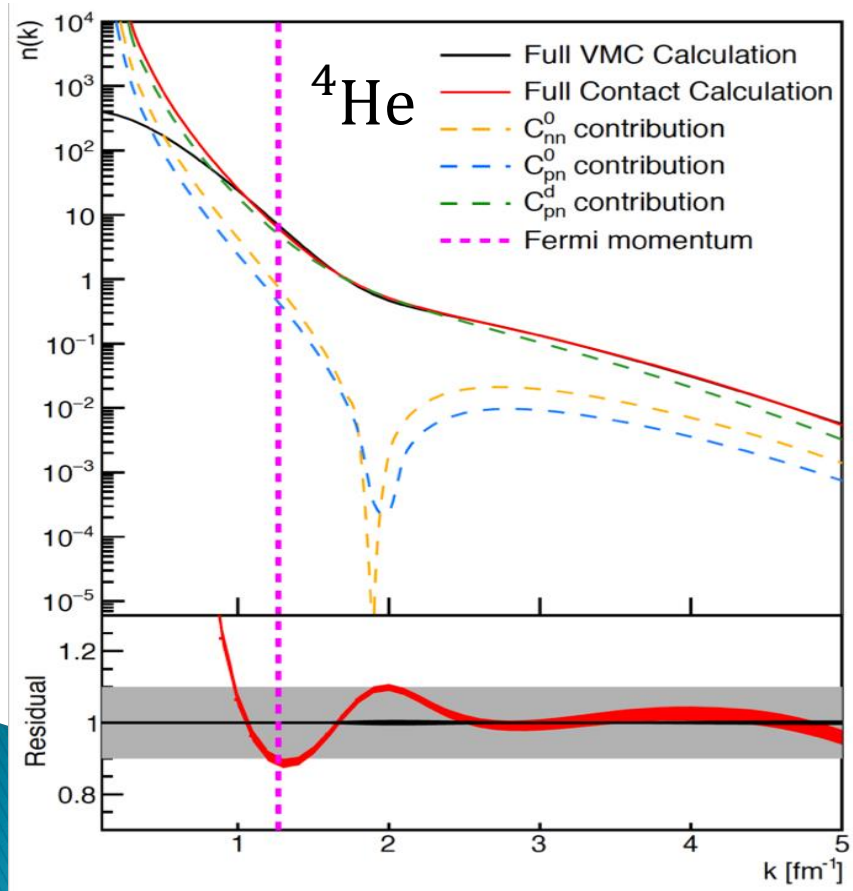
pp/np



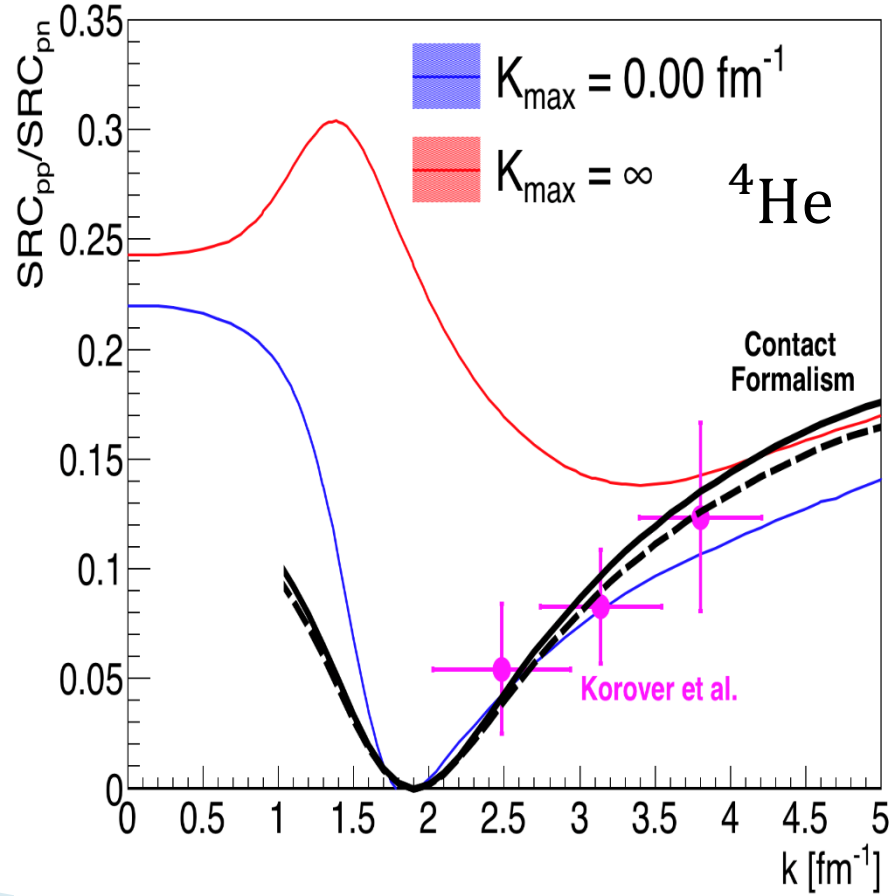
Extracting the contacts

$$n_p(k) \xrightarrow[k\to\infty]{} C^d_{pn}\left|\varphi^d_{pn}(k)\right|^2 + C^0_{pn}\left|\varphi^0_{pn}(k)\right|^2 + 2C^0_{pp}\left|\varphi^0_{pp}(k)\right|^2$$

$n_p(k)$



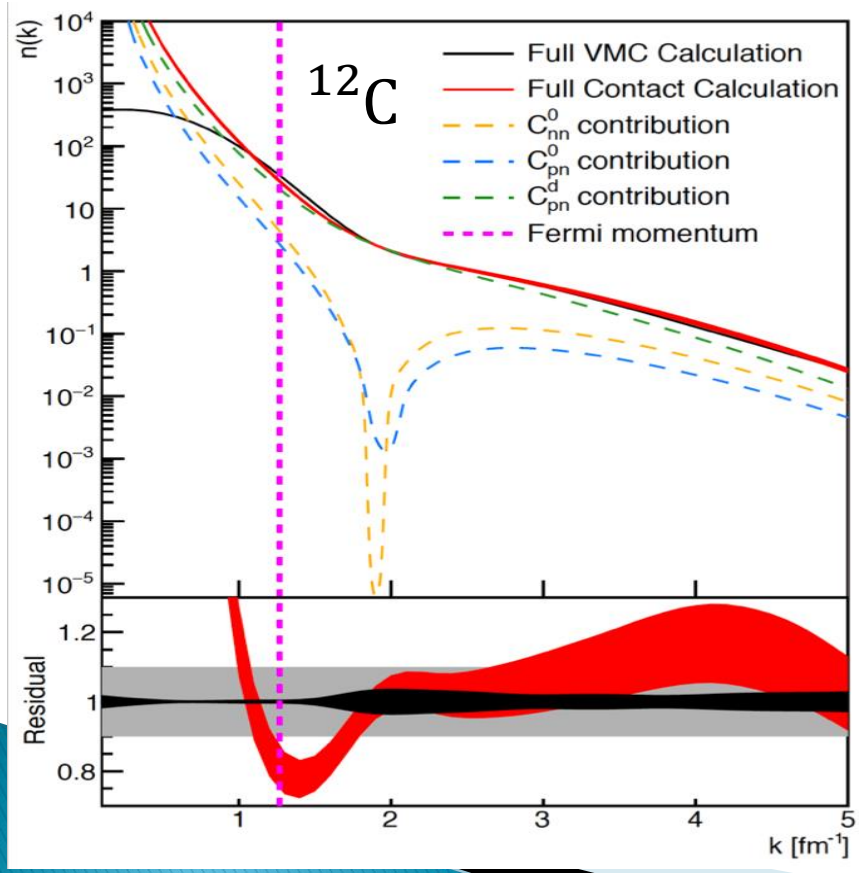
pp/np



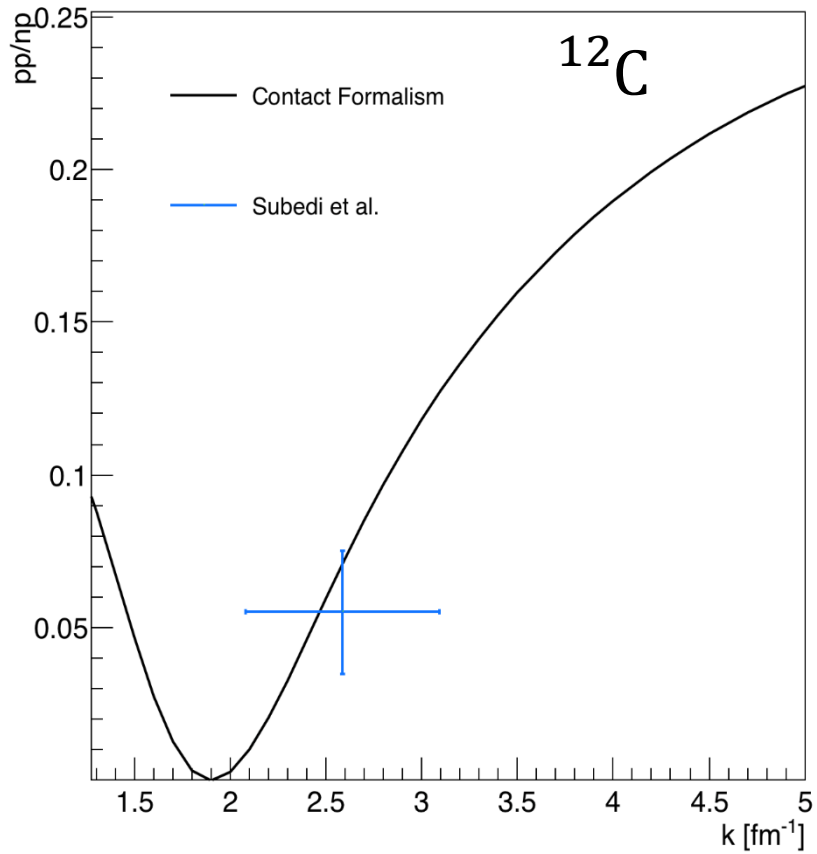
Extracting the contacts

$$n_p(k) \xrightarrow[k\to\infty]{} C^d_{pn}\left|\varphi^d_{pn}(k)\right|^2 + C^0_{pn}\left|\varphi^0_{pn}(k)\right|^2 + 2C^0_{pp}\left|\varphi^0_{pp}(k)\right|^2$$

$n_p(k)$



pp/np



Counting the SRCs (symmetric nuclei)

$$n_p(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2 + 2C_{pp}^{0}\left|\varphi_{pp}^{0}(k)\right|^2$$

Normalization: $\int_{k_F}^{\infty}\left|\varphi_{ij}^{\alpha}\right|^2 d^3k = 1$

$$\%SRC \equiv \frac{1}{Z}\int_{K_F}^{\infty} n_p(\boldsymbol{k})d^3k = \frac{1}{Z}\left[C_{pn}^{d} + C_{pn}^{0} + 2C_{nn}^{0}\right]$$

Counting the SRCs

^{4}He → Total number of pairs:

pp – 1 np–4

Counting the SRCs

^{4}He → Total number of pairs:
pp – 1 np–4

	C^0_{pp}/Z (%)	C^0_{pn}/Z (%)	C^d_{pn}/Z (%)
k–space	0.65 ± 0.03	0.69 ± 0.03	12.3 ± 0.1

Non–combinatorial isospin symmetry (T=1)

Neutron–proton dominance

Counting the SRCs

^{4}He → Total number of pairs:
pp – 1 np–4

	C^0_{pp}/Z (%)	C^0_{pn}/Z (%)	C^d_{pn}/Z (%)
k–space	0.65 ± 0.03	0.69 ± 0.03	12.3 ± 0.1

Non–combinatorial isospin symmetry (T=1)

Neutron–proton dominance

No need in S=1, T=1 channels

Counting the SRCs

^{4}He → Total number of pairs:
pp – 1 np–4

	C^0_{pp}/Z (%)	C^0_{pn}/Z (%)	C^d_{pn}/Z (%)	%SRCs
k–space	0.65 ± 0.03	0.69 ± 0.03	12.3 ± 0.1	14.3%

Counting the SRCs

^{4}He → Total number of pairs:
pp – 1 np–4

	C^0_{pp}/Z (%)	C^0_{pn}/Z (%)	C^d_{pn}/Z (%)	%SRCs
k–space	0.65 ± 0.03	0.69 ± 0.03	12.3 ± 0.1	14.3%
r–space	0.567 ± 0.004		11.61 ± 0.03	13.3%

Similar results are obtained for all the available nuclei in the VMC data

The nuclear contact relations

- **This work**

  R. Weiss, R. Cruz-Torres, N. Barnea E. Piasetzky and O. Hen, arXiv:1612.00923 [nucl-th]

- **Momentum distributions**

  *R. Weiss, B. Bazak, N. Barnea, PRC **92**, 054311 (2015)*

  *M. Alvioli, CC. Degli Atti, H. Morita, PRC **94**, 044309 (2016)*

- **The Levinger constant**

  *R. Weiss, B. Bazak, N. Barnea, PRL **114**, 012501 (2015)*

  *R. Weiss, B. Bazak, N. Barnea, EPJA **52**, 92 (2016)*

- **Electron scattering**

  *O. Hen et al., PRC **92**, 045205 (2015)*

- **Symmetry energy**

  *BJ. Cai, BA. Li, PRC **93**, 014619 (2016)*

- **The Coulomb sum rule (and a review)**

  *R. Weiss, E. Pazy, N. Barnea, Few-Body Systems **58**, 9 (2017)*

- **The EMC effect**

  JW. Chen, W. Detmold, J. E. Lynn, A. Schwenk, arxiv 1607.03065 [hep-ph] (2016)

and more…

Summary

Two-body momentum distribution for $k > 4\ \mathrm{fm}^{-1}$

Two-body coordinate density for $r < 1\ \mathrm{fm}$

Extracting the contacts

Full details on SRCs for $k > k_F$

Summary

Two-body momentum distribution for $k > 4\ \mathrm{fm}^{-1}$

Two-body coordinate density for $r < 1\ \mathrm{fm}$

Extracting the contacts

Full details on SRCs for $k > k_F$

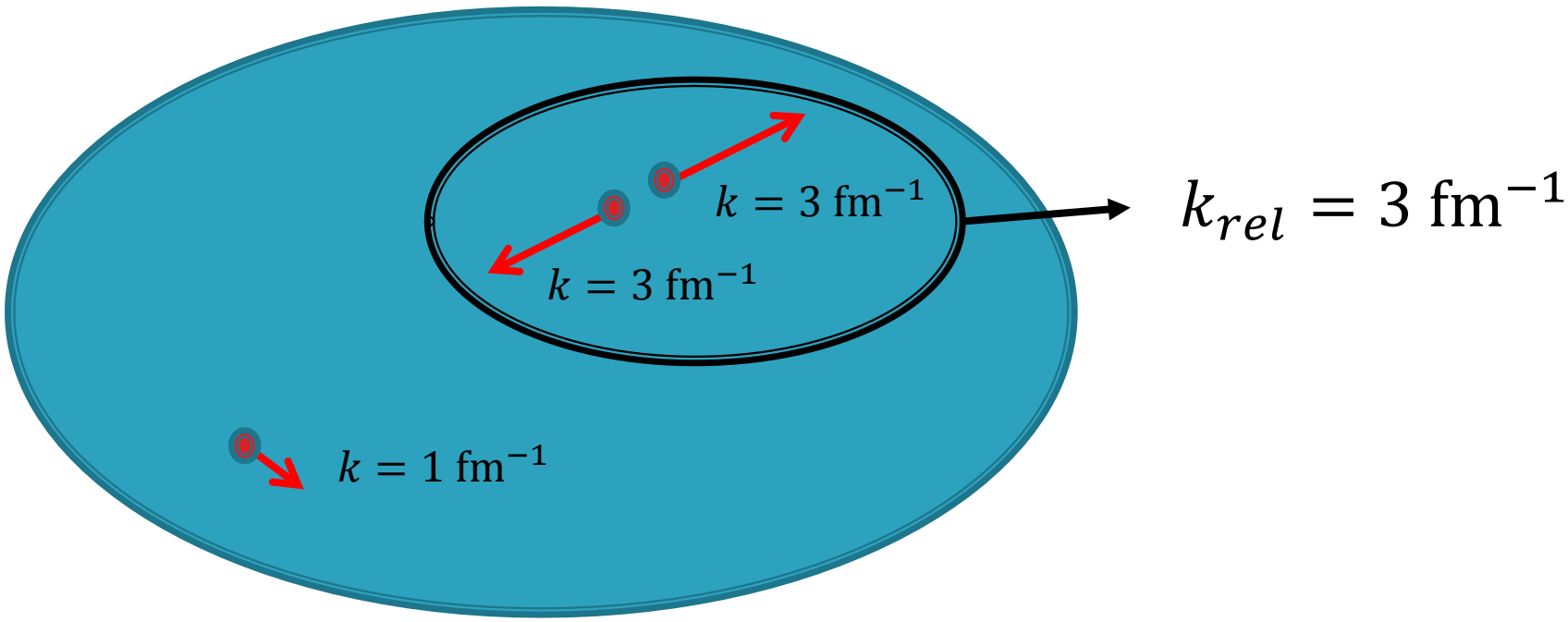
- np dominance & pp/np
- Isospin symmetry
- %SRCs
- Main (L, S, J, T) channels
- 1B momentum distribution

Questions?

Back up

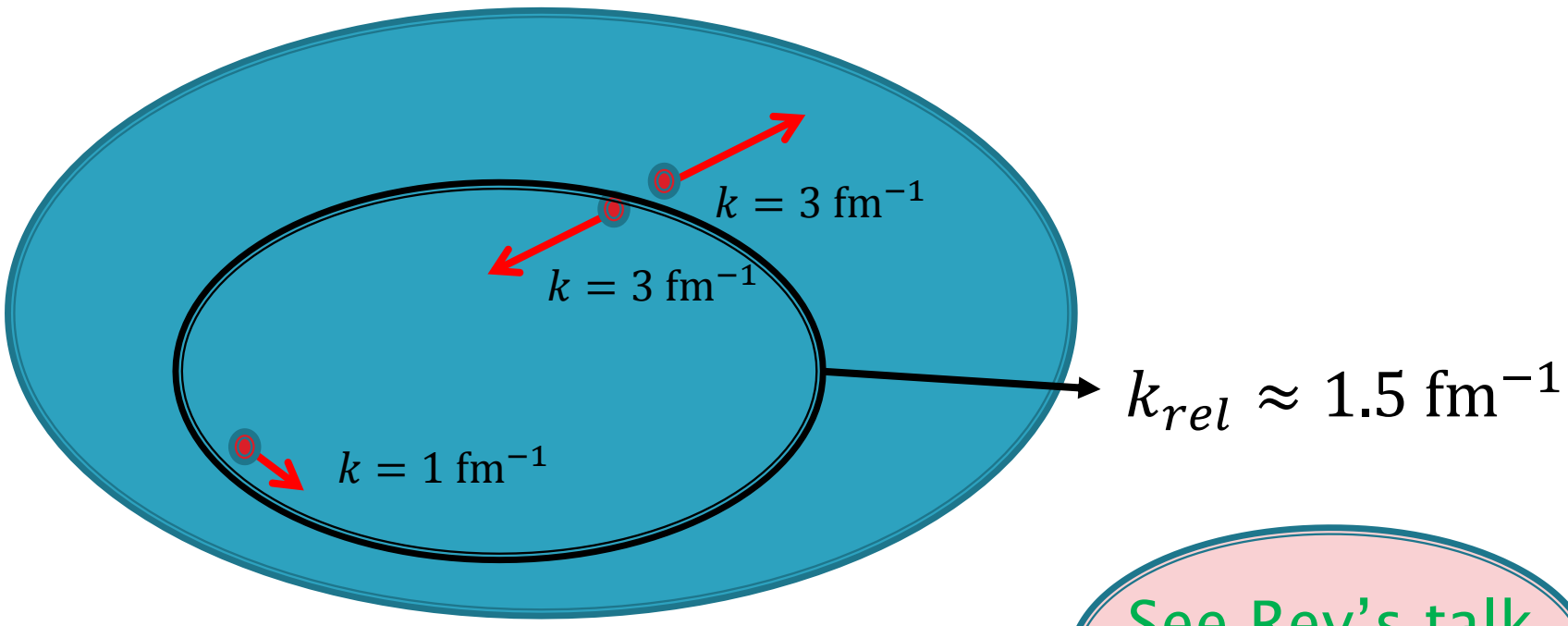
The two-body momentum

- The two-body momentum distribution can be "contaminated" by non-correlated pairs



The two-body momentum

- The two-body momentum distribution can be "contaminated" by non-correlated pairs

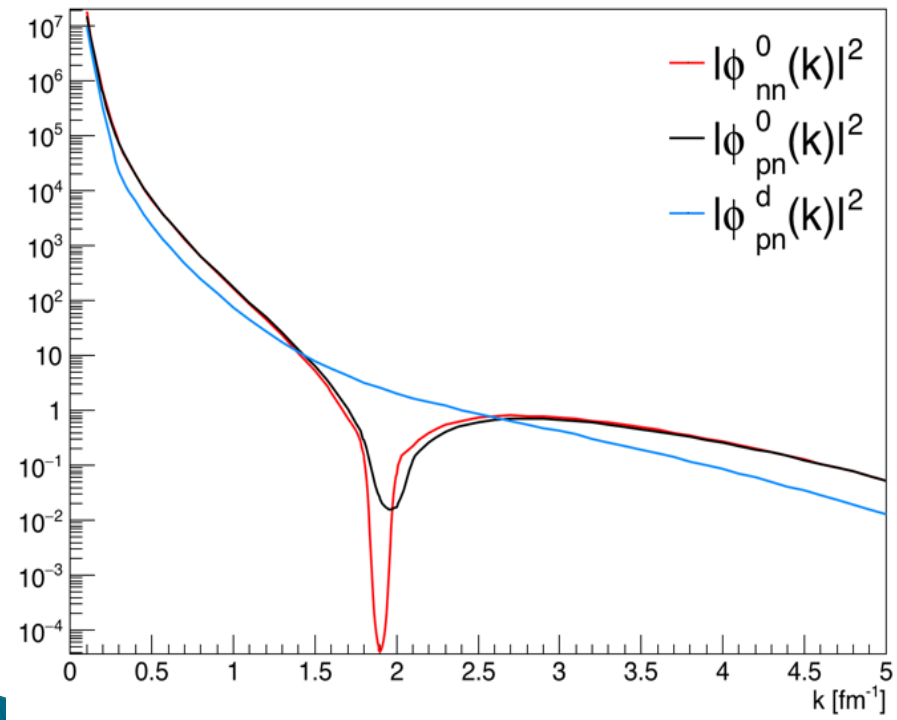


See Rey's talk next session

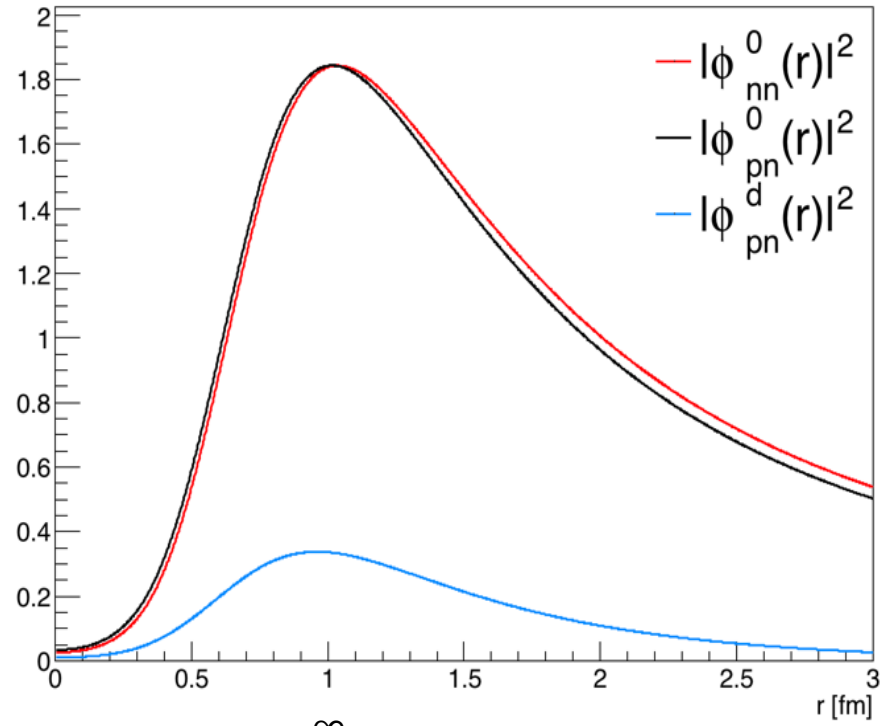
The universal functions

- Using the AV18 potential: $|\varphi_{ij}^{\alpha}|^2$

Momentum space



Coordinate space



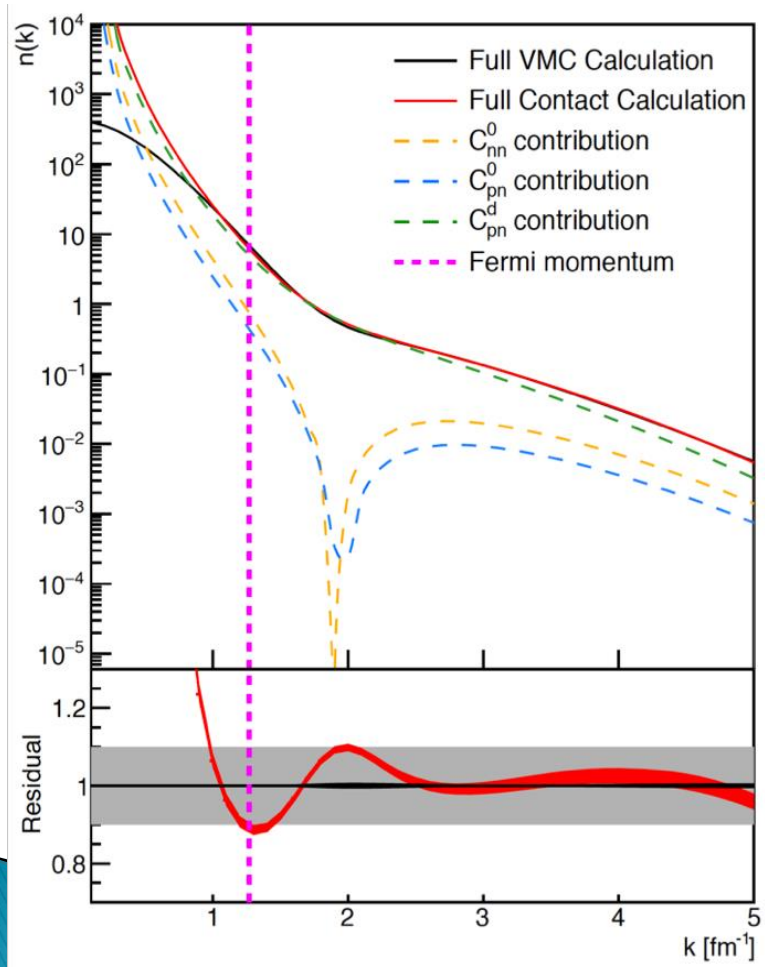
$$\int_{k_F}^{\infty} \left|\varphi_{ij}^{\alpha}\right|^2 d^3k = 1$$

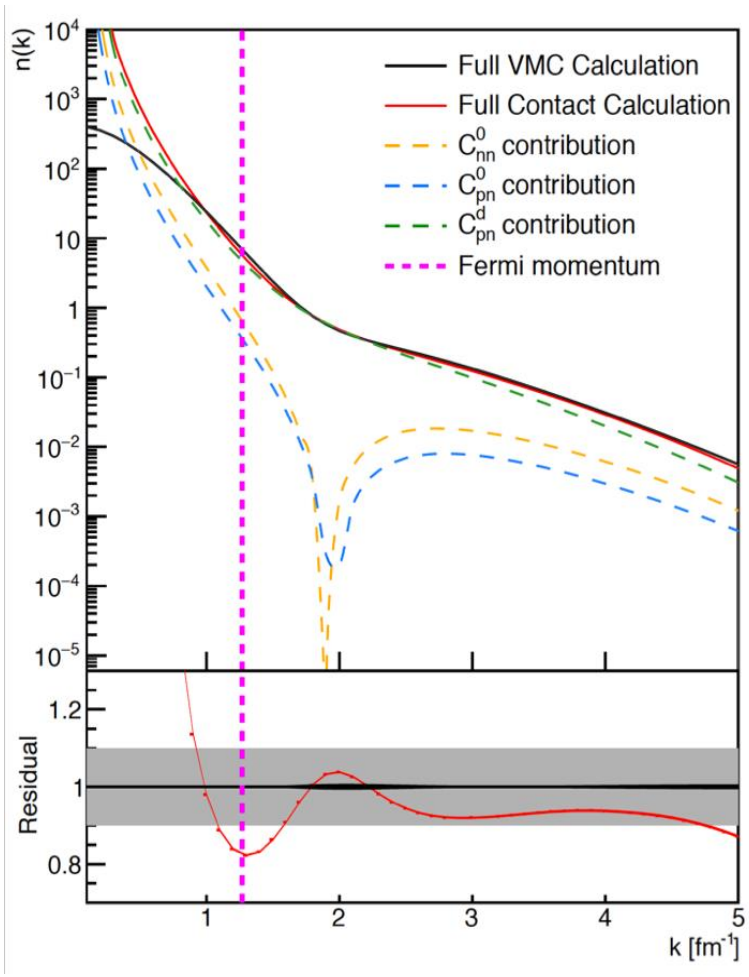
One-body reconstruction

Momentum extraction

^{4}He

Coordinate extraction



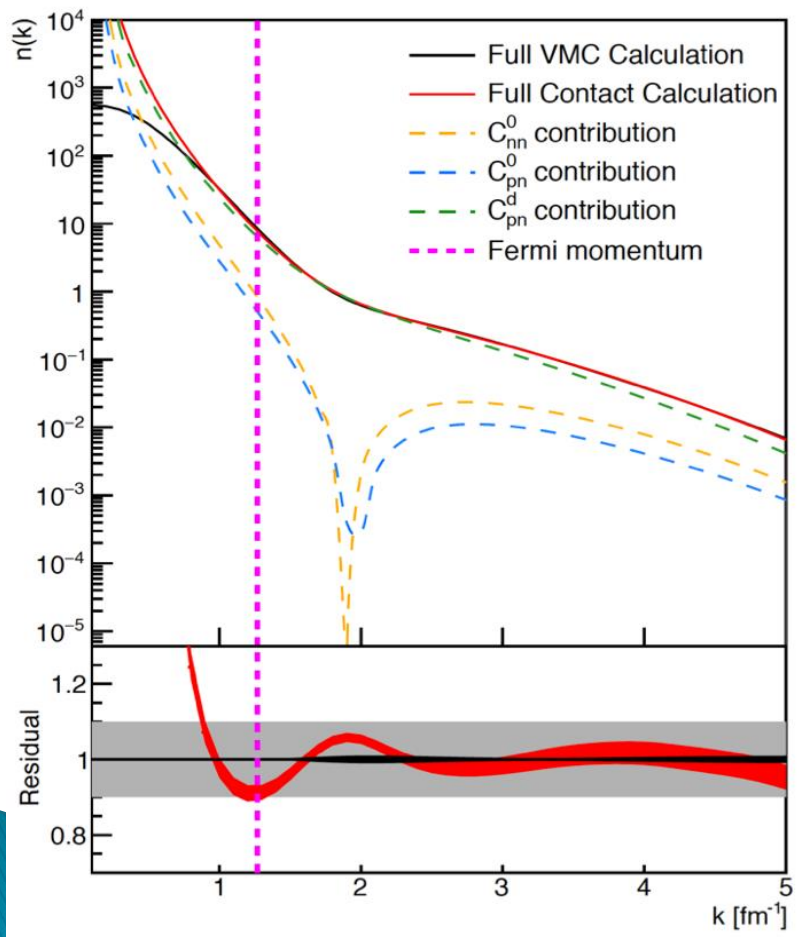


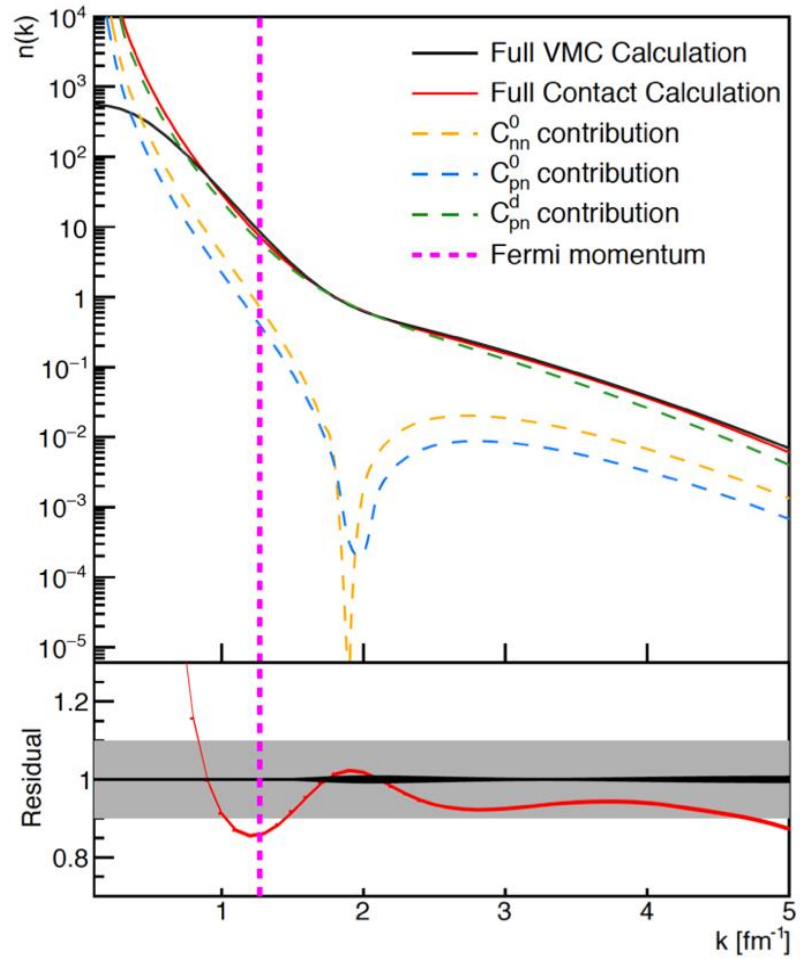
One-body reconstruction

Momentum extraction

^{6}Li

Coordinate extraction



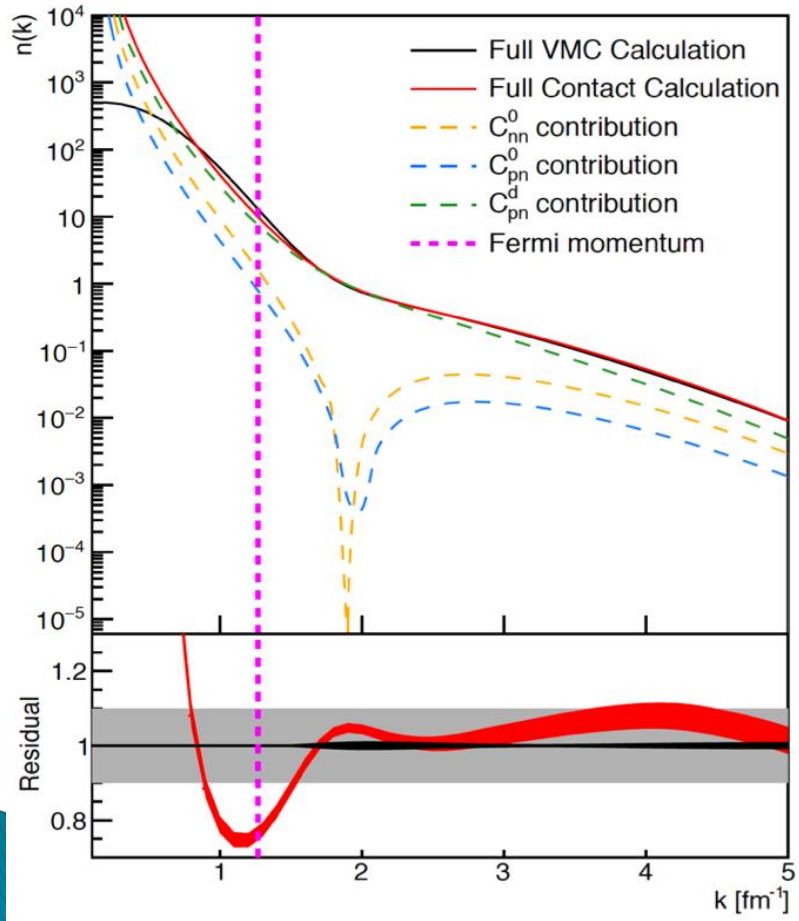


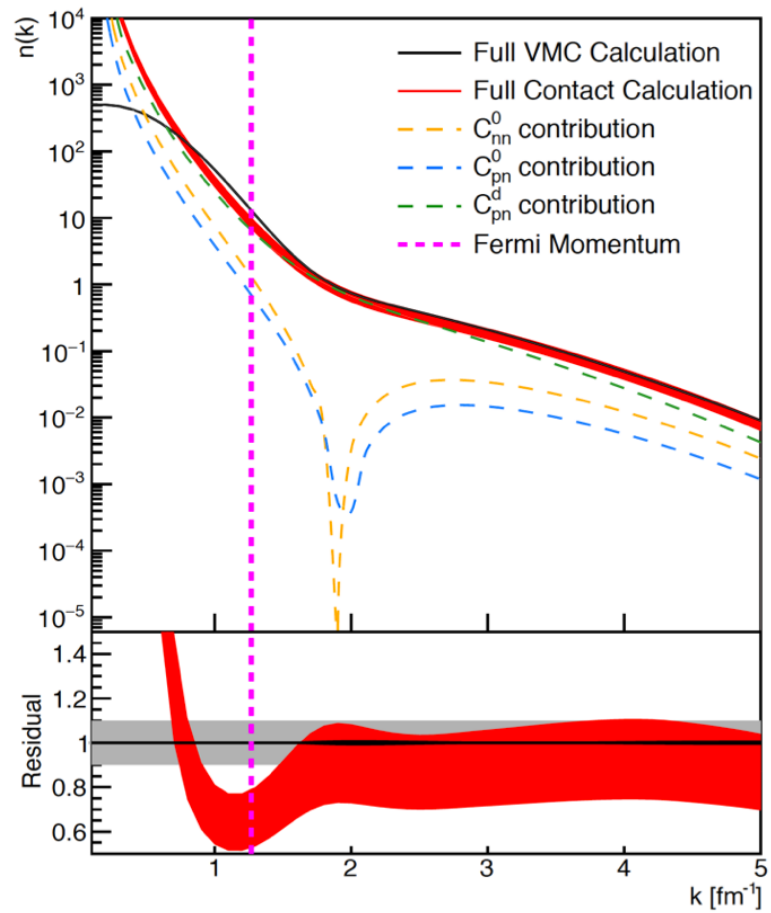
One-body reconstruction

Momentum extraction

^{7}Li

Coordinate extraction



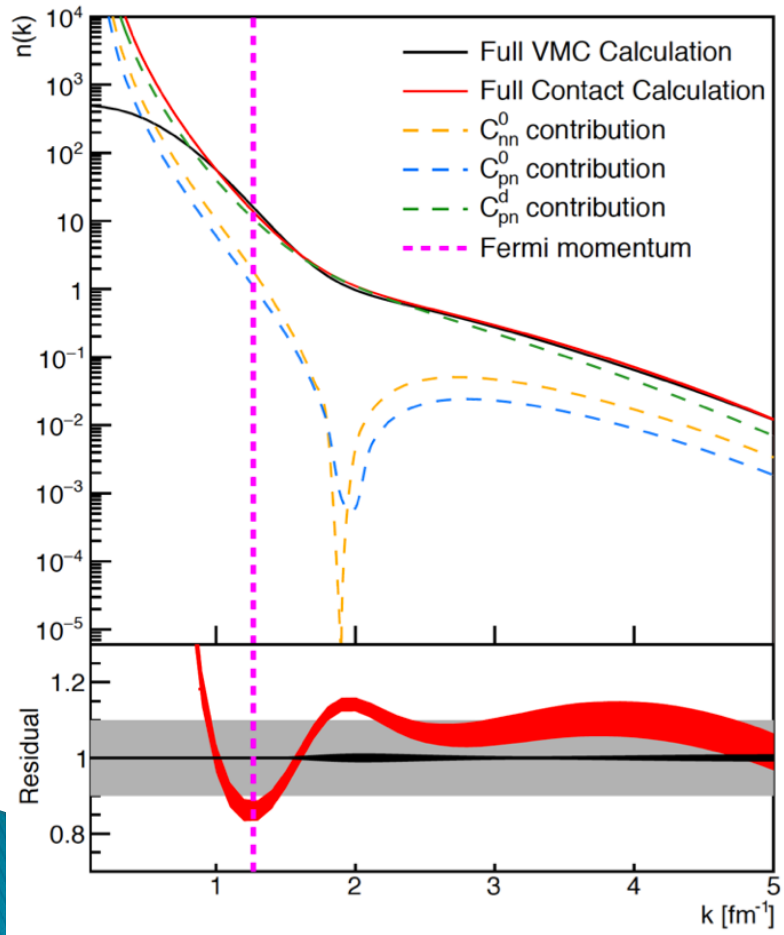


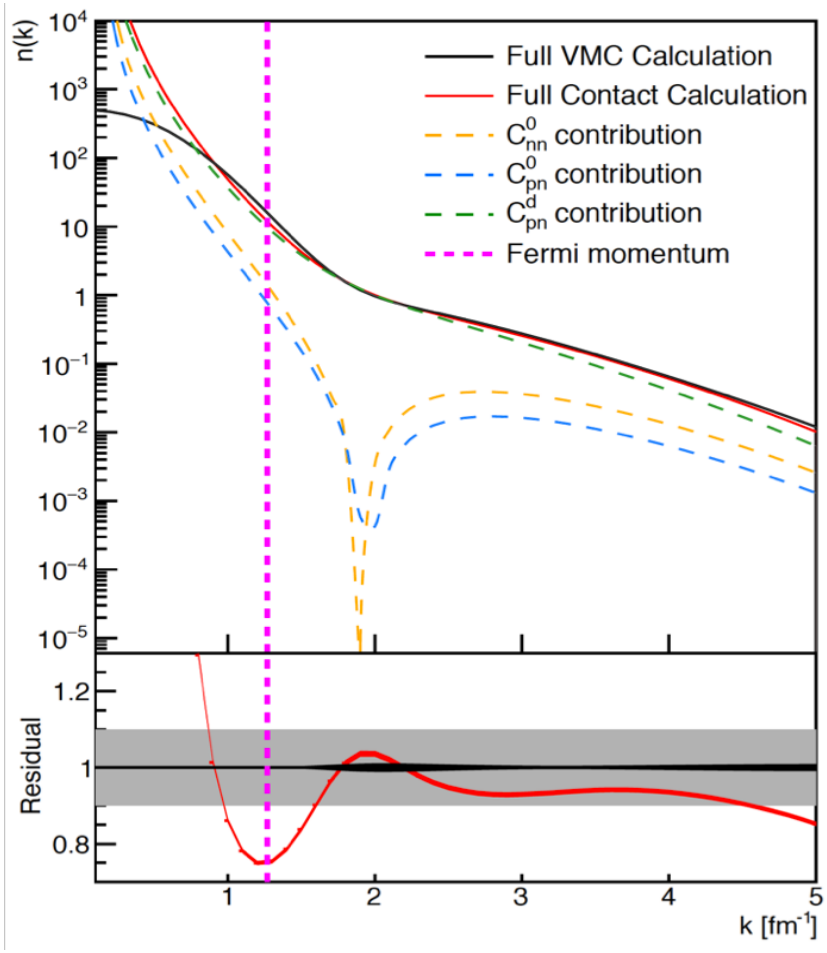
One-body reconstruction

Momentum extraction

^{8}Be

Coordinate extraction





One-body / Two-body

- The one-body momentum distribution:

$$n_n(\boldsymbol{k}) = N \int \prod_{l \neq n} d\boldsymbol{k}_l \left| \widetilde{\Psi}(\boldsymbol{k}_1, \dots, \boldsymbol{k}_n, \dots, \boldsymbol{k}_A) \right|^2$$

- The two-body momentum distribution:

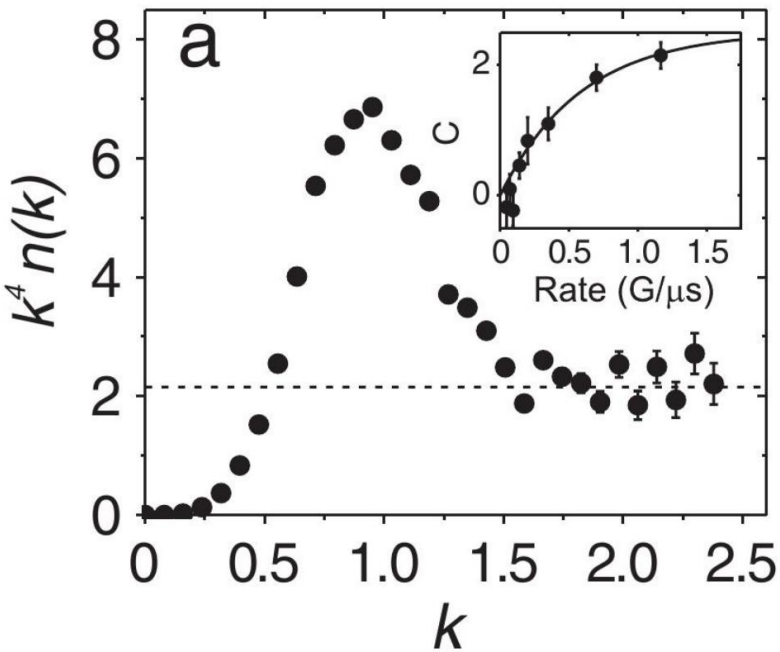
$$F_{ij}(\boldsymbol{k}_{rel}) = N_{ij} \int d\boldsymbol{K}_{ij} \prod_{l \neq i,j} d\boldsymbol{k}_l \left| \widetilde{\Psi}\left(\boldsymbol{k}_1, \dots, \boldsymbol{k}_i, \dots, \boldsymbol{k}_j, \dots, \boldsymbol{k}_A\right) \right|^2$$

$$ij = pp, nn, np$$

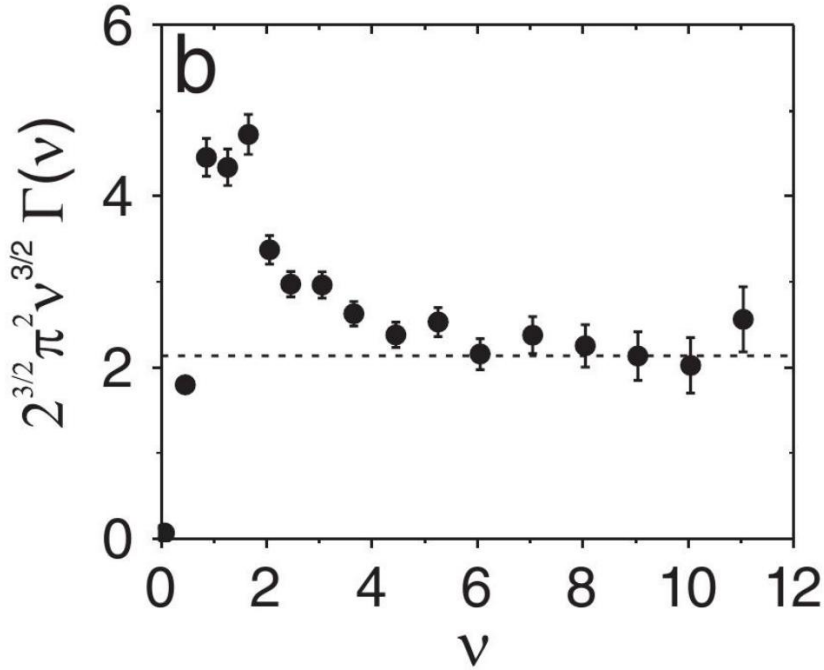
N_{ij} – number of ij pairs

The atomic contact

Momentum distribution



RF line shape



J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, Phys. Rev. Lett. 104, 235301 (2010)