

Stability of Phases and Interacting Particle Systems

Nick Crawford; The Technion

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Based on joint work w/ W. De Roeck

Setup

- ▶ $\Omega = \{-1, 1\}^{\mathbb{Z}^d}$ or $\Omega_N = \{-1, 1\}^{\Lambda_N}$ with $\Lambda_N = [-N, N]^d$.
- ▶ Finite Range Markov process $\sigma_t \in \Omega$ or $\in \Omega_N$.
No Conservation Laws!
- ▶ Semigroup $e^{t\mathcal{L}} : C(\Omega) \rightarrow C(\Omega)$.
Generator \mathcal{L} acting on local functions:

$$\mathbb{E}_\sigma[f(\sigma_t)] = e^{t\mathcal{L}} \cdot f(\sigma).$$

Main Example of Interest: Ising Glauber Dynamics

1. $-H_N(\sigma|\eta) =$

$$1/2 \sum_{|x-y|=1, x, y \in \Lambda_N} \sigma_x \sigma_y + \sum_{|x-y|=1, x \in \Lambda_N, y \in \Lambda_N^c} \sigma_x \eta_y.$$

2. Spin Flip Operator:

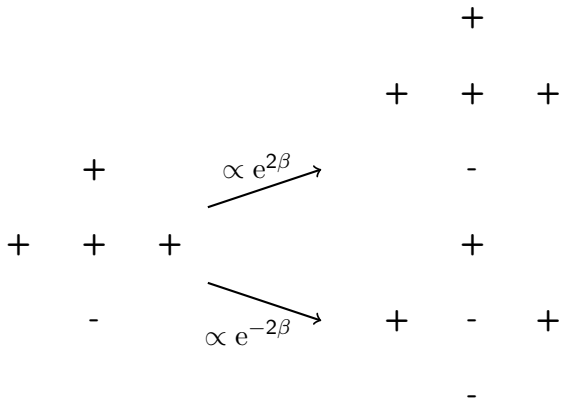
$$\sigma_y^x = \begin{cases} -\sigma_x & \text{if } y = x \\ \sigma_y & \text{o/w.} \end{cases}$$

3. For $\sigma \in \Omega_N$,

$$c_x(\sigma|\eta) = (1 + \exp(\beta[H(\sigma^x|\eta) - H(\sigma|\eta)]))^{-1}$$

$$\text{and } \mathcal{L}_N^\eta \cdot f(\sigma) = \sum_{x \in \Lambda_N} c_x(\sigma|\eta) [f(\sigma^x) - f(\sigma)].$$

Similar on Ω



Invariant Measures

1. ν a measure on Ω . Invariance:

$$\mathbb{E}_\nu[f(\sigma_t)] = \mathbb{E}_\nu[f(\sigma_0)]$$

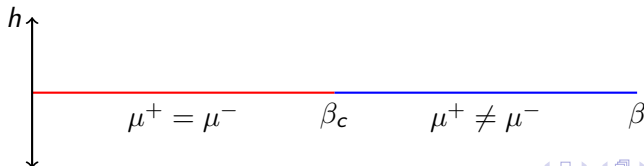
2. For Glauber, β fixed:

All Gibbs measures invariant. Weak limits of

$$\mu_{N,\beta}^\eta(\sigma) \propto e^{-\beta H_N(\sigma|\eta)}.$$

Others?

Structure of Gibbs measures:



Perturbations and Stability of Phases

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- ▶ **The Big Question:** Suppose $\bar{\sigma}_t$ has generator $\bar{\mathcal{L}}$ "close" to \mathcal{L} . Is the Uniqueness/Multiplicity of stationary measures preserved?
- ▶ Caution: Glauber with Nonzero External Fields.
- ▶ **Current Understanding for Glauber:**
 1. $\beta < \beta_c$ or $h \neq 0$ **Yes. (C-DR '17)**
 2. $\beta > \beta_c$ and $h = 0$ **Don't Know. Very Interesting.**

Examples:

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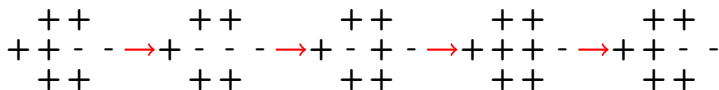
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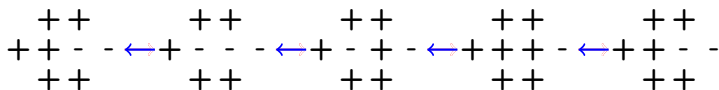
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Previous Related Work

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Everything is true:

1. Unique invariant measure, is Gibbs
Exponential decay of all truncated correlations.
2. Extends to quantum systems, i.e. Linbladans (Yarotsky '05, C, DR, Schütz '15).
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- ▶ In 1d, uniqueness phase for attractive N.N. particle systems (Grey '82).

Weak Spatial Mixing (W.S.M.)

Let σ_t be attractive, F.R. with $\mathcal{L} \cdot f(\sigma) = \sum_x c_x(\sigma)[f(\sigma^x) - f(\sigma)]$.

On Ω_N gives **family** of processes: for $\eta \in \Omega, \sigma \in \Omega_N$.

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$$\nu_N^+(\sigma(0)) - \nu_N^-(\sigma(0)) \leq Ce^{-cN}.$$

Theorem (Martinelli-Oliveri '94)

If σ_t satisfies W.S.M., ! stationary ν satisfies

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Big problem in Non-reversible setting.....

Main Results

- ▶ $c_x(\sigma)$ rates of F.R. attractive σ_t .
- ▶ $\bar{\sigma}_t$ F.R. 'perturbed' w/ rates $d_x(\sigma)$. **No need for attractivity!**
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Corollary

First conclusion holds for Ising Glauber in entire uniqueness phase.

Information (Non)-Percolation

Individual Reservoirs: Fix $\epsilon, \beta > 0$ and $|\beta_x - \beta| < \epsilon$.

$$d_x(\sigma) = (1 + \exp(\beta_x[H(\sigma^x) - H(\sigma)]))^{-1}$$

The Basic Coupling:

Indep., to each x attach

1. Rate 1 Poisson process $N_t(x)$.
2. An i.i.d. sequence $U_{x,i}$ of $[0, 1]$ uniform variables.

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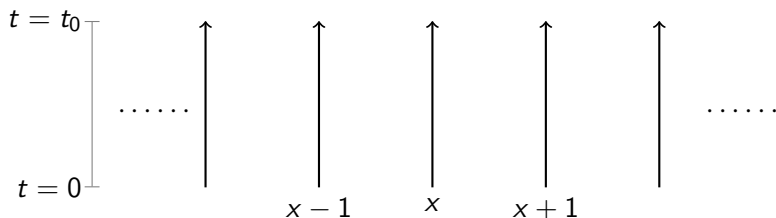
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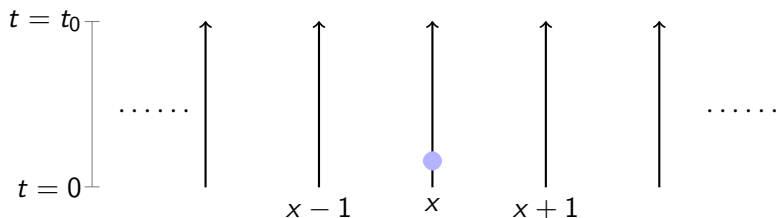
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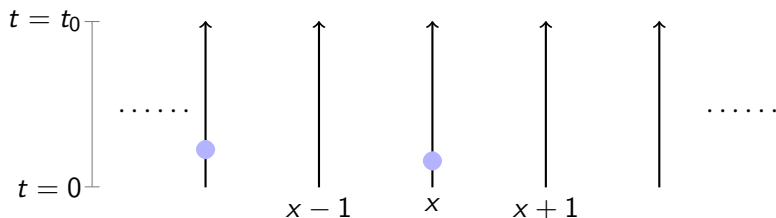
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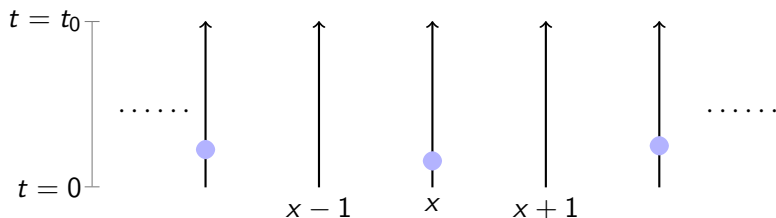
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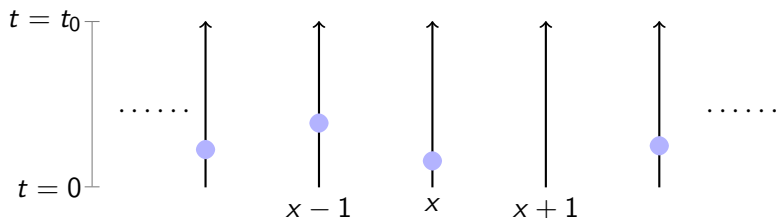
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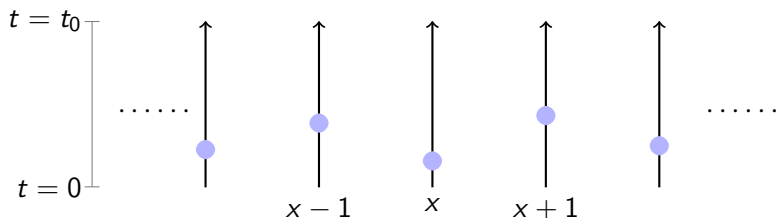
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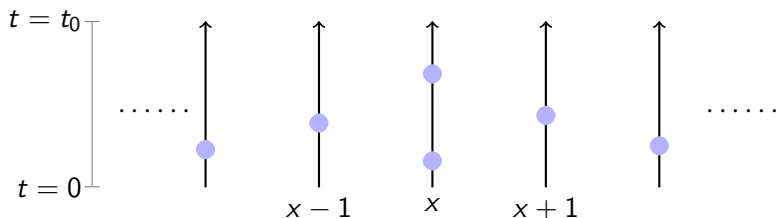
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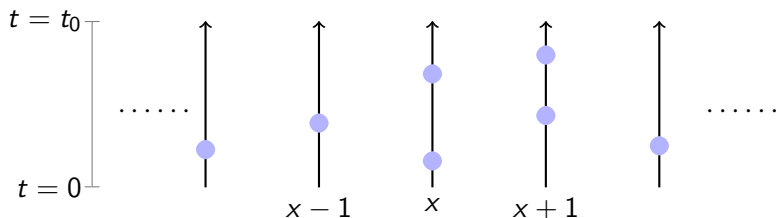
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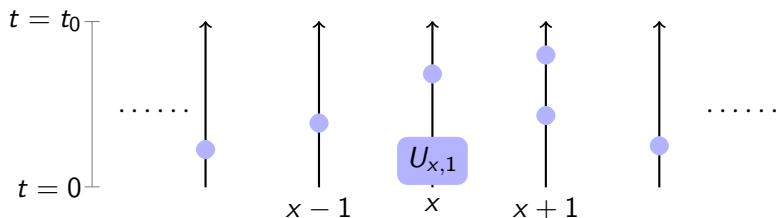
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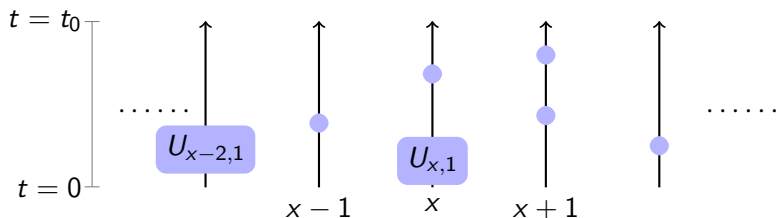
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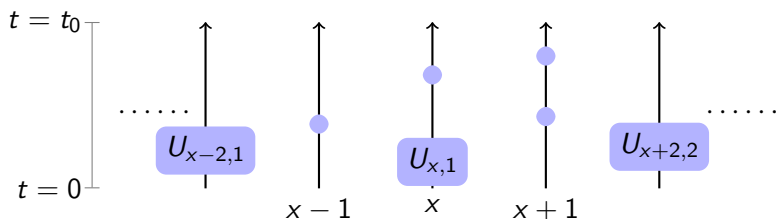
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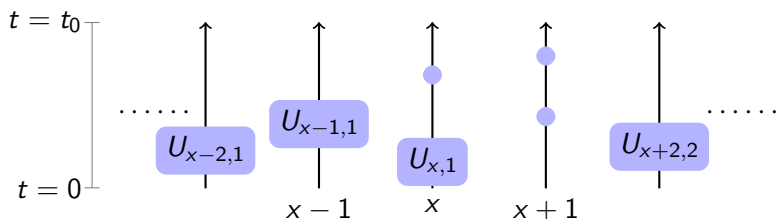
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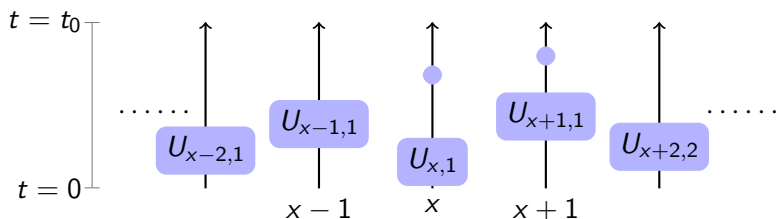
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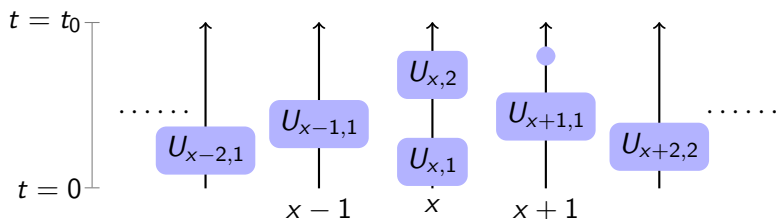
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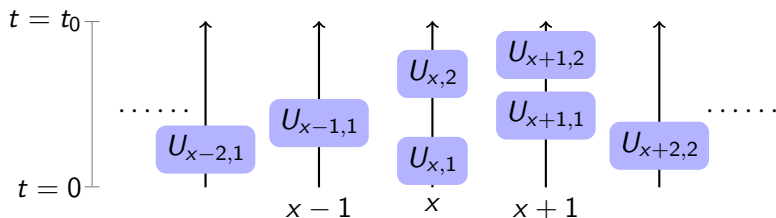
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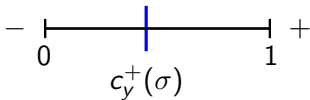
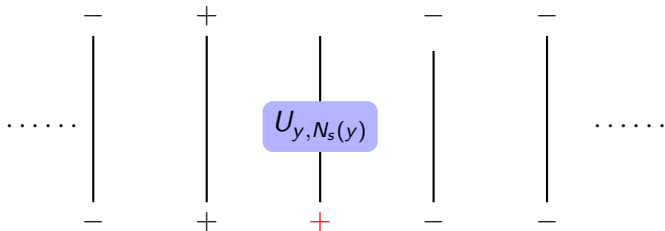
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$t = s + \Delta t$

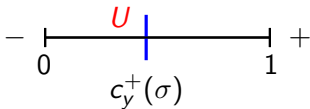
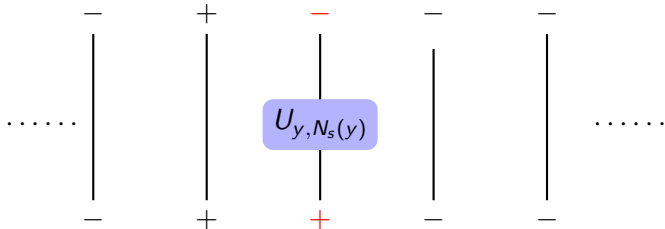
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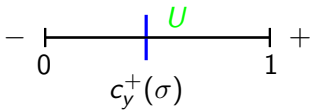
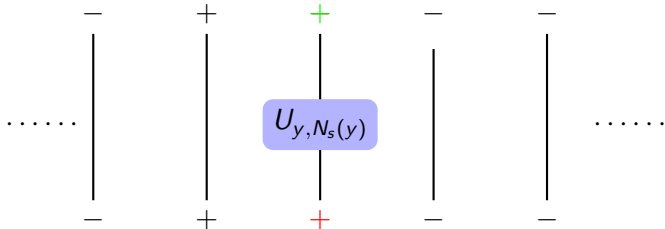
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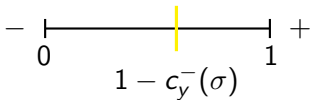
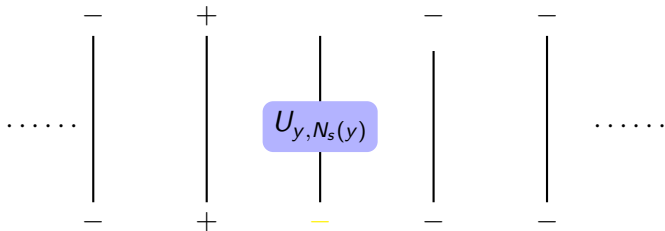
$t = s$



If $\sigma_s(y) = -$

$t = s + \Delta t$

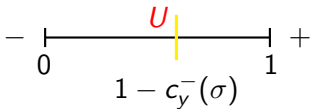
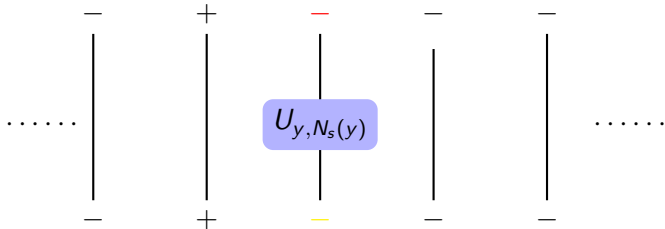
$t = s$



If $\sigma_s(y) = -$

$t = s + \Delta t$

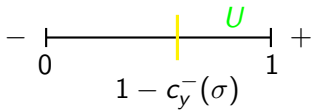
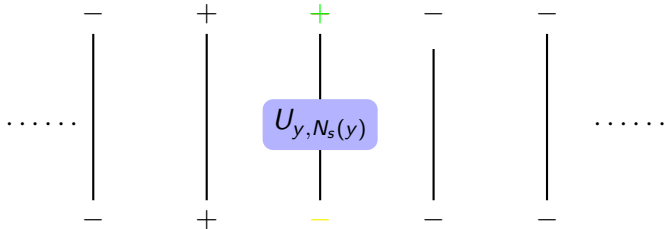
$t = s$



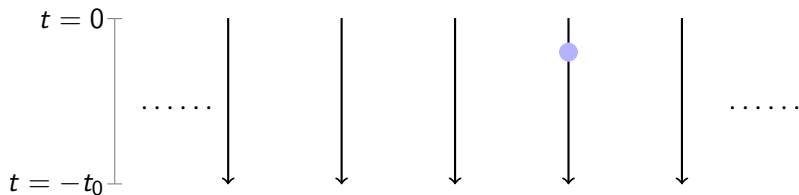
If $\sigma_s(y) = -$

$t = s + \Delta t$

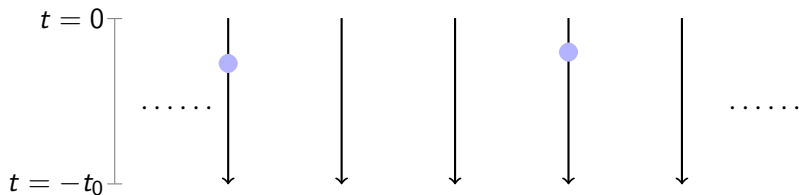
$t = s$



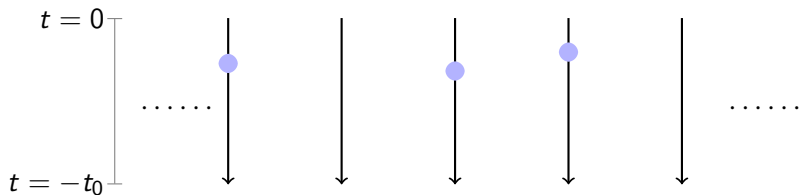
Coupling from Past (Propp, Wilson '96)



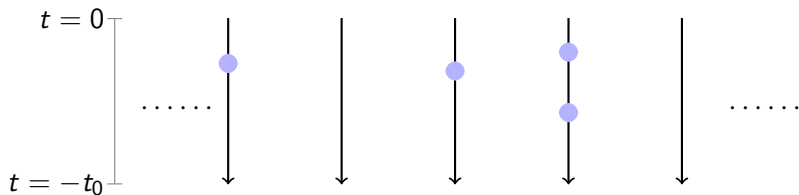
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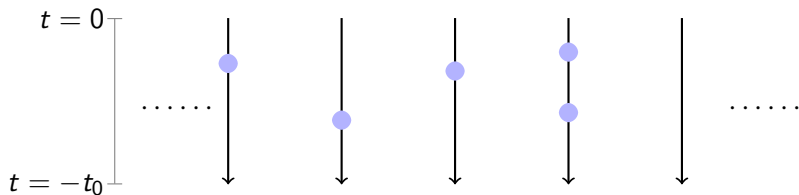
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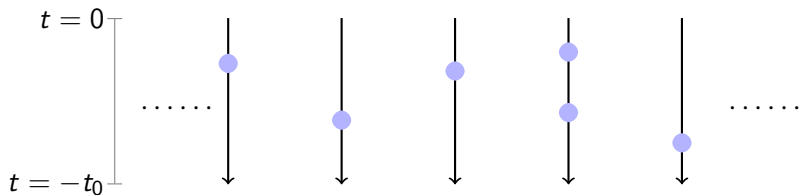
Coupling from Past (Propp, Wilson '96)



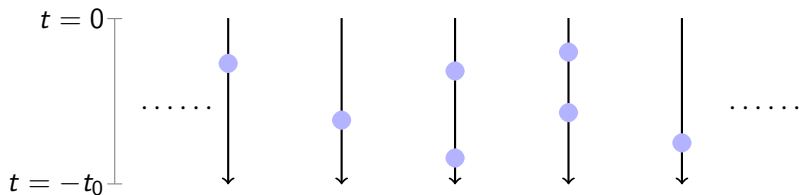
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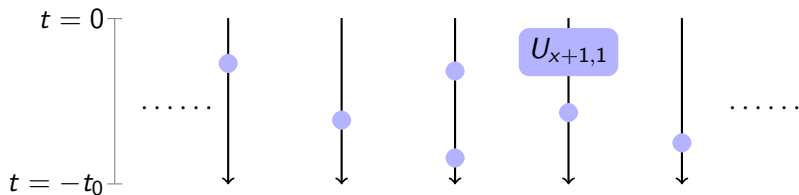
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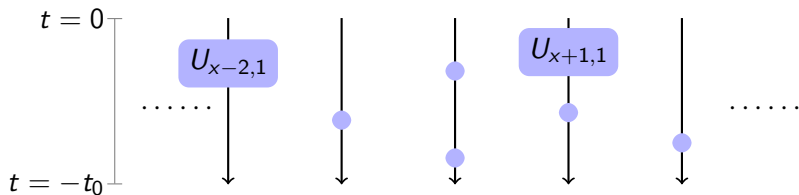
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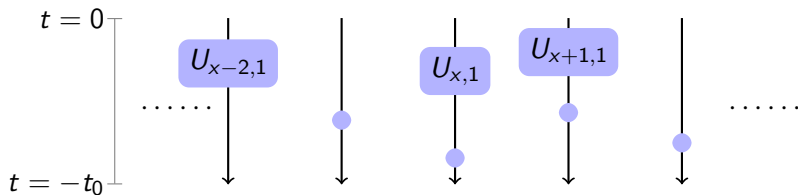
Coupling from Past (Propp, Wilson '96)



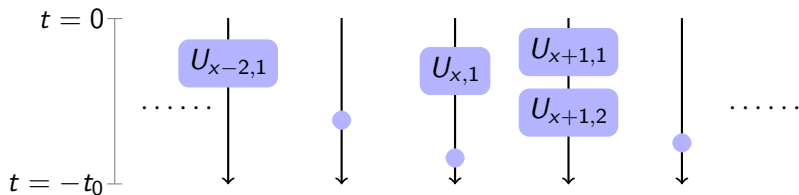
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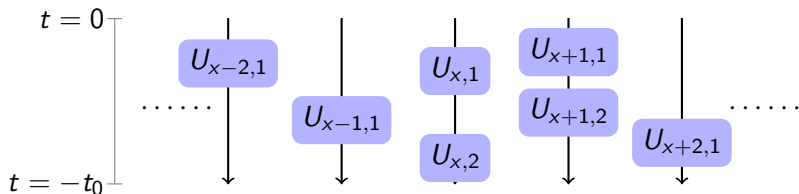
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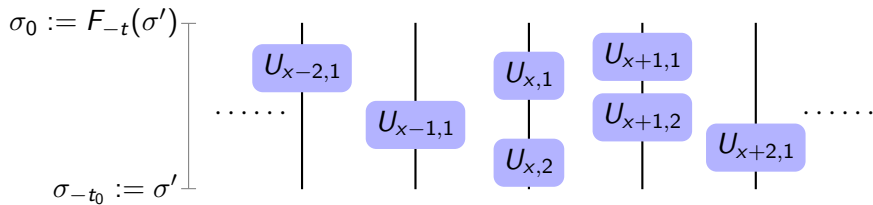


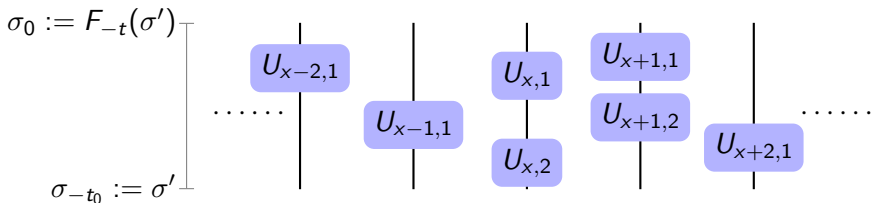
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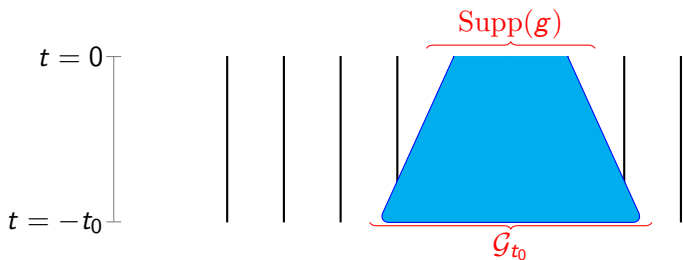
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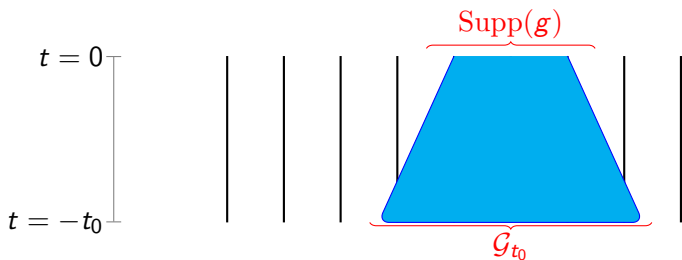


- ▶ Given N 's and U 's, σ_0 **deterministic** function of $\sigma_{-t} = \sigma'$ that is $\sigma_0 = F_{-t}(\sigma')$.



► For any g local, $\mathcal{G}_t := \text{Supp}_{\mathbb{Z}^d} g \circ F_{-t}$

$$\mathcal{G}_t = \emptyset \Leftrightarrow g \circ F_{-t} = \text{cnst},$$



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$$\sup_{\sigma_0} |\mathbb{E}_{\sigma_0}[g(\sigma_t)] - \nu(g)| \leq \mathbb{P}(\mathcal{G}_t \neq \emptyset).$$

Attractivity \Rightarrow

$$\mathbb{P}(\mathcal{G}_t \neq \emptyset) = \mathbb{E}_+[g(\sigma_t)] - \mathbb{E}_-[g(\sigma_t)].$$

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Lemma (Lubetsky-Sly '13)

For equilibrium Ising Glauber dynamics,

$$\mathbb{E}_+[\sigma_t(0)] - \mathbb{E}_-[\sigma_t(0)] = \mathbb{P}(\mathcal{G}_t = \emptyset)$$

In particular $\mathbb{P}(\mathcal{G}_t = \emptyset) \leq Ce^{-ct}$.

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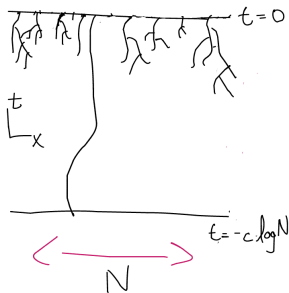
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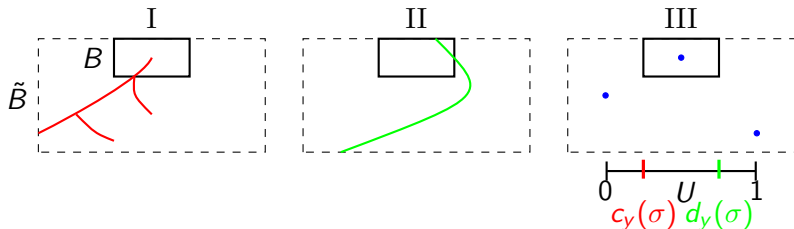
Space-Time non-Percolation

Course Graining: Fix $r, N \in \mathbb{N}$

$$B_0 = \{0, 1, \dots, rN - 1\}^d \times (0, N],$$

$$B_{\mathbf{n}} = B_0 + (rNk, N\ell) \quad \text{for } \mathbf{n} = (k, \ell) \in \mathbb{Z}^d \times \mathbb{Z}.$$

Bad Boxes:



A box is good o/w.

Lemma

For N large enough, and ϵ small enough (depending on N)

$$\mathbb{P}(B_n \text{ is bad}) \leq e^{-cN}.$$

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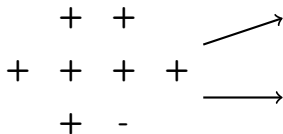
Consider $\mathbf{X} := (\mathbf{1}\{B_n \text{ is bad}\})_{n \in \mathbb{Z}^{d+1}}$.

Lemma

For N large enough and ϵ small enough, \mathbf{X} is dominated by subcritical site percolation (*-connected sense).

Open Questions

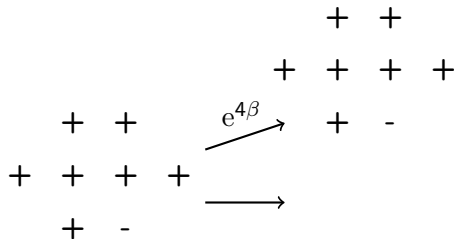
1. In uniqueness phase, is \bar{v} Gibbs? (Redig et. al.)
2. Is attractivity essential in uniqueness phase? e.g.



3. Stability of Coexistence.

Open Questions

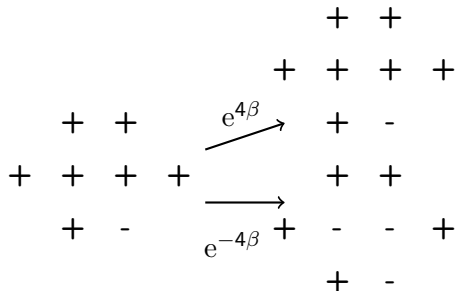
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